Non-linear Vibration Harvesting

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N.i.P.S Laboratory
Noise in Physical Systems

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Energy harvesting in Erice:

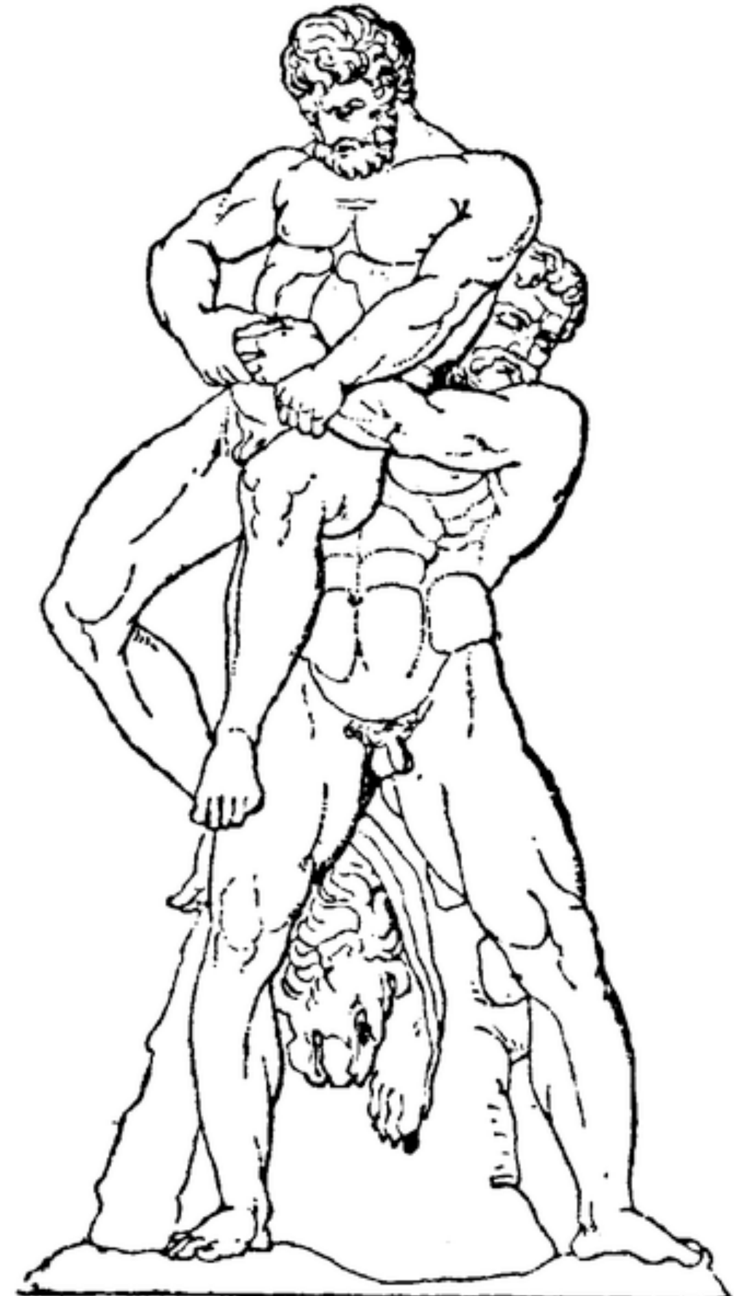
Erice is the right place where speaking about energy at least according with one legend:

Here took place a famous fight challenge between Eryx (or Erice, a demigod son of the goddess Aphrodite, king of the ancient town in the 13^{th} century BC, a famous and strong boxer) and his friend Heracles (or Hercules, son of Zeus) a divine hero, the greatest of the Greek heroes.

Eryx put us as a prize his reign and Heracles the cows of the monstrous Gerion.

Heracles won killing Eryx and Erice became a Heraclean town. This is why Erice became later a greek colony.

So Erice has a strong connection with the symbol of men's energy (almost a linear system…)



Energy harvesting in Erice:

Moreover in Erice there was an important Aphrodite temple (because Eryx was her son), and another legend tells that Aeneas during his trip to found Rome arrived in Trapani where his father Anchises died.

Aeneas (son of Anchises and Aphrodite) decided to bury his father in Erice, close to her temple.

Aphrodite represents love and all about beauty and feminine vital energies.

So Erice has even a strong connection with the symbol of women energy (a strongly non-linear system…)

Vibration energy harvesting

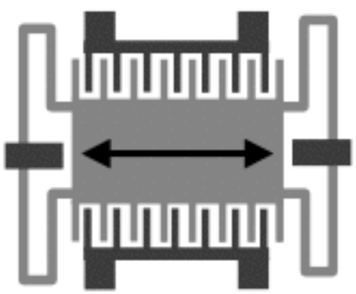
Four main transduction mechanisms

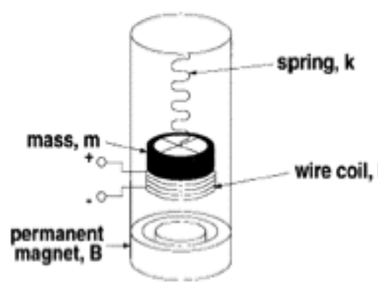
Capacitive: geometrical variations induce voltage difference

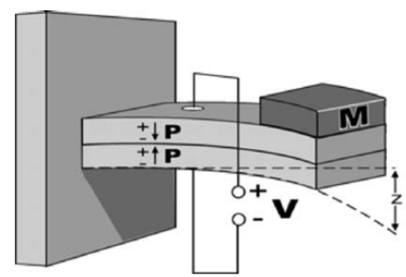
Piezoelectric: dynamical strain is converted into voltage difference.

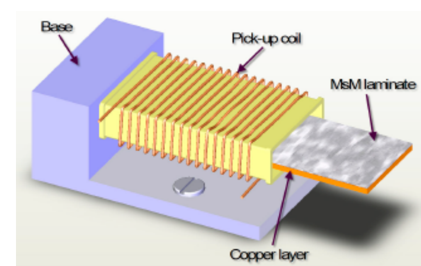
Inductive: dynamical oscillations of magnets induce electric current in coils

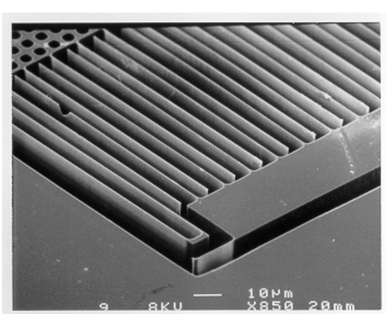
Magnetostrictive: stress produces a variable magnetic field that induces a current in an adjacent conductive coil.

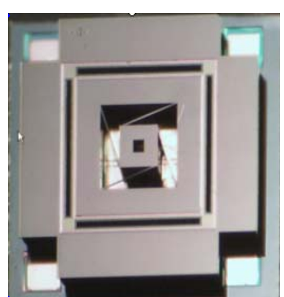


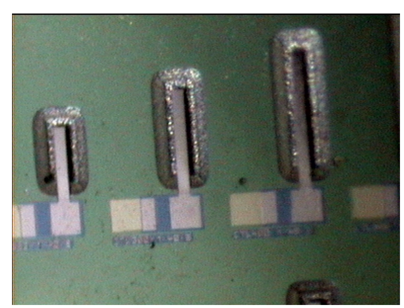


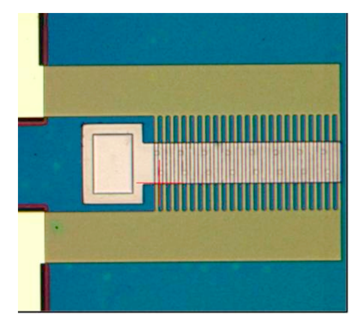




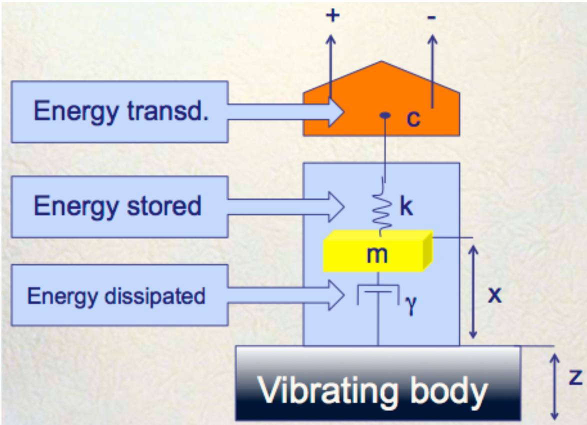








Vibrations harvesting: the model



$$m\ddot{x} = -\frac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z$$

Force due to the energy stored

Dissipative force

Reaction force due to the transduction mechanism

Input force

$$\begin{cases} m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - c(x,V) + \zeta_z \\ \dot{V} = F(\dot{x},V) \end{cases}$$

Details depend on the physics…

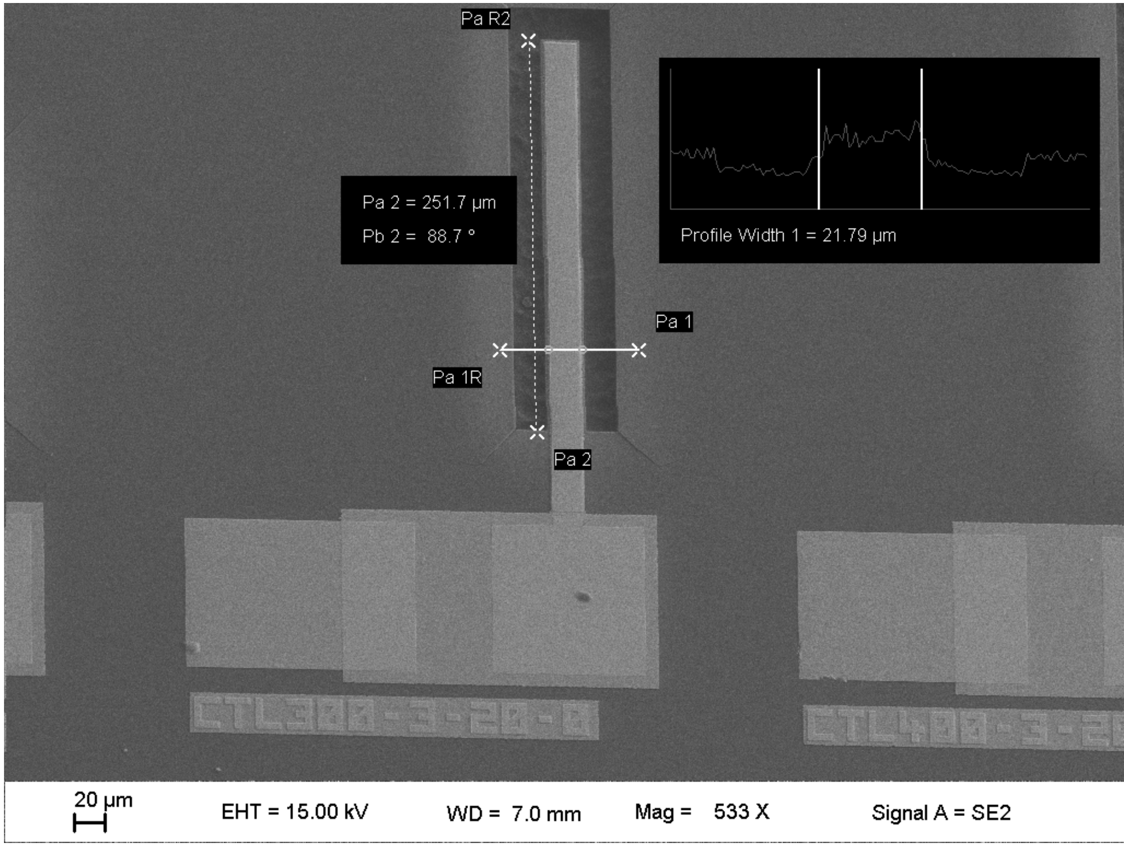
Equations that link the vibration-induced displacement with the Voltage

Vibrations harvesting: the transduction mechanism

We will focus on **Piezoelectricity** because for practical reasons has the best coupling factor.

Capacitive: is more easy to scale down but you have to pay a debt: it needs a bias voltage

Inductive and Magnetostrictive: are more difficult to be scaled down and have a lower coupling factor



Vibrations harvesting: the model for piezo

$$
\begin{cases}
m\ddot{x} = -\dfrac{dU(x)}{dx} - \gamma\dot{x} - K_v V + \zeta_z \\[2ex]
\dot{V} = K_c\dot{x} - \dfrac{1}{\tau_p} V
\end{cases}
$$

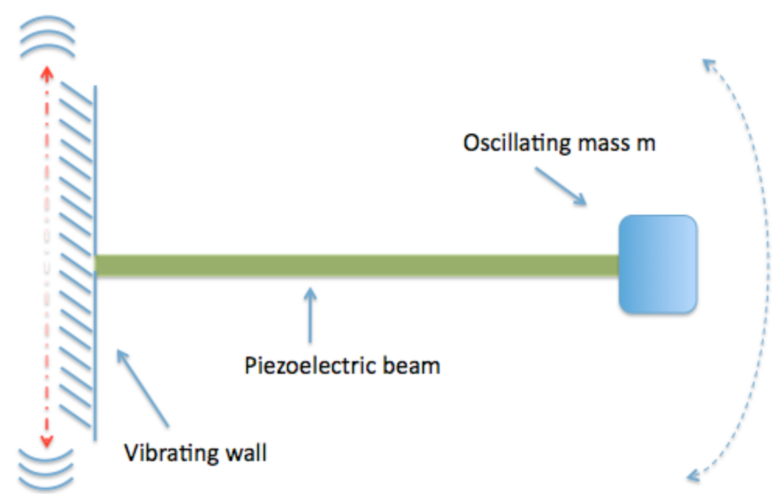
That for a beam are:

$$K_v = \frac{K_{eff} d_{31} a}{2 t_p k_1}$$

$$K_c = \frac{t_p d_{31} Y_p^E k_1}{a \varepsilon_p}$$

The Physics of piezo materials

Now we focus on ζ_z



The random character of kinetic energy

ζ_z Represents the vibrational stochastic force

Random vibrations / noise

Thermal noise (NOT POSSIBLE AT EQUILIBRIUM!!!)
Acoustic noise
Seismic noise
Ambient noise (wind, pressure fluctuations, …)
Man made vibrations (human motion, machine vibrations,…)

All different for intensity, spectrum, statistics

How can we harvest them ?

Linear system

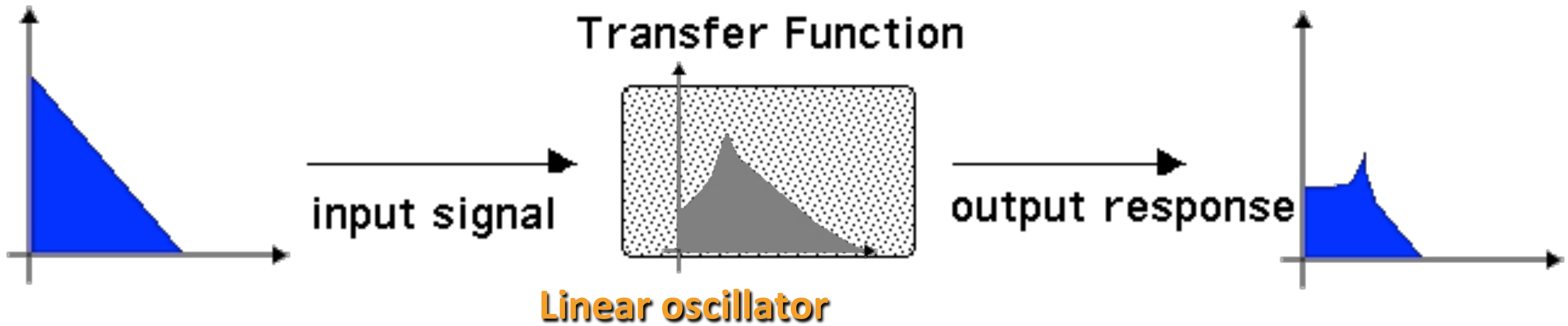
If a linear system is considered: $U(x) \approx x^2$

1) There exist a simple math theory to solve the equations
2) They have a resonant behaviour (resonance frequency)
3) They can be “easily” realized with cantilevers and pendula

Linear system

X(s) i.e. ambient energy *H(s)* *Y(s)* i.e. output energy



The transfer function is a math function of the frequency, in the complex domain, that can be used to represent the performance of a linear system and can act as a filter…

$$Y(s) = H(s)\, X(s)$$

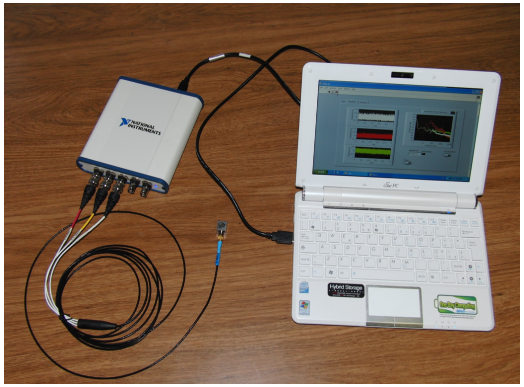
For a linear system the transfer function presents one or more peeks corresponding to the resonace frequencies.

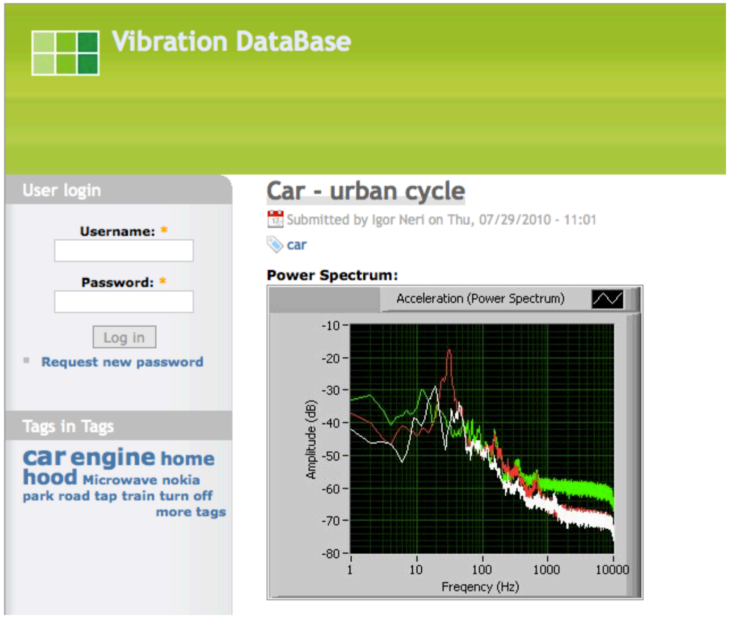
A linear system is the most performing if its resonance frequency is where the incoming energy is abundant…

This is a serious limitation when you want to build a small energy harvesting system working in a real environment…

For two main reasons:

The frequency spectrum of available vibrations instead of being sharply peaked at some frequency is usually very broad.



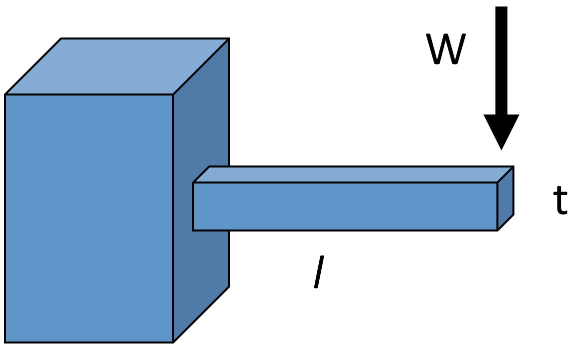


Accelerometer:
- Tri axial
- Bandwidth from 0.4Hz to 10kHz
- ±50g

DAQ:
- 102.4 kS/s five simultaneous channel
- 4 channels with software-selectable IEPE signal conditioning
- USB powered

Signal presentation:
- Description
- Power spectrum
- Statistical data
- Time series download (only for authorized users)

The frequency spectrum of available vibrations is particularly rich in energy in the low frequency part… and it is very difficult, if not impossible, to build small low-frequency resonant systems…



Resonant frequency ~ $[\mathrm{s}^{-1}]$

- MEMS cantilever 100 × 3 × 0.1 µm^3, f_0=12 kHz
- NEMS cantilever 0.1 × 0.01 × 0.01 µm^3, f_0=1.2 GHz

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \delta = \frac{Wl^3}{3EI} \qquad k = \frac{W}{\delta} = \frac{3EI}{l^3}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{3EI}{Ml^3}} = \frac{1}{2\pi}\sqrt{\frac{Ewt^3}{4Ml^3}} = \frac{t}{4\pi l^2}\sqrt{\frac{E}{\rho}}$$

From the model for a linear oscillator:

The voltage transfer function is:

$$|H(\omega)| = \frac{1}{m}\frac{\omega k_c}{\sqrt{\omega^2\left(\frac{k+\frac{\gamma}{\tau}+k_c k_v}{m}-\omega^2\right)^2+\left(\left(\frac{\gamma}{m}+\frac{1}{\tau}\right)\omega^2-\frac{k}{m\tau}\right)^2}}$$

or considering: $\omega_0 = \sqrt{\dfrac{k+\frac{\gamma}{\tau}+k_c k_v}{m}}$

and: $\omega_1 = \sqrt{\dfrac{k}{\gamma\tau + m}}$

$$\Rightarrow \quad |H(\omega)| = \frac{1}{m}\frac{\omega k_c}{\sqrt{\omega^2\left(\omega^2-\omega_0^2\right)^2+\left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2}}$$

if $\omega^2\left(\omega^2-\omega_0^2\right)^2 > \left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2$

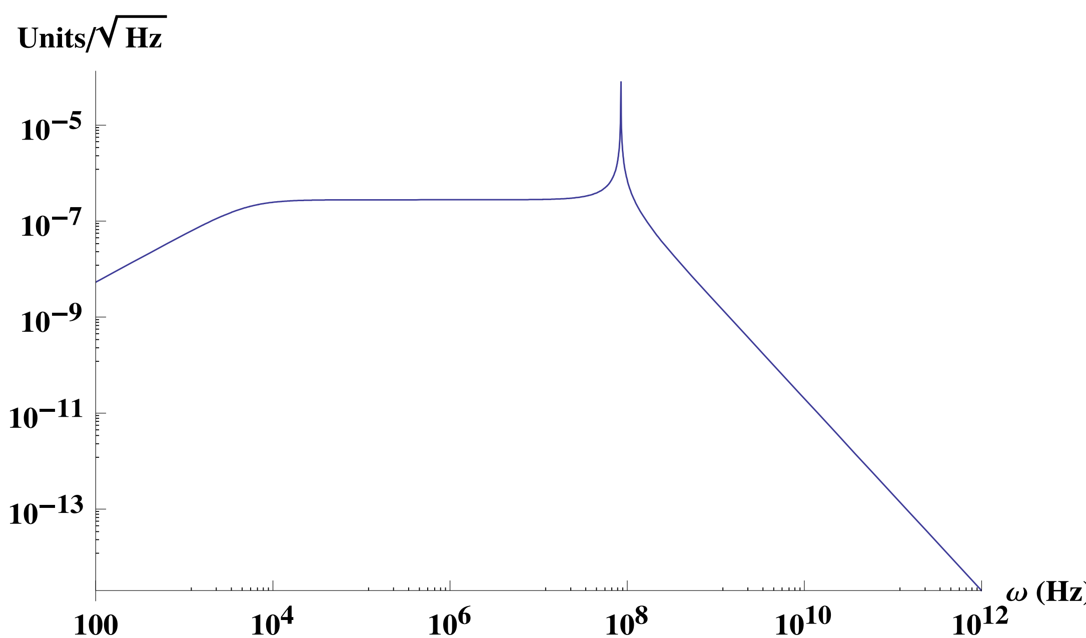
the resonance frequency is $\omega_0 = \sqrt{\dfrac{k+\frac{\gamma}{\tau}+k_c k_v}{m}}$

where $|H(\omega)|_{\max} = \dfrac{\omega_0 k_c \tau}{(\gamma\tau+m)\left(\omega_0^2-\omega_1^2\right)^2}$

if $\omega^2\left(\omega^2-\omega_0^2\right)^2 < \left(\frac{\gamma\tau+m}{m\tau}\right)^2\left(\omega^2-\omega_1^2\right)^2$

the resonance frequency is $\omega_1 = \sqrt{\dfrac{k}{\gamma\tau + m}}$

where $|H(\omega)|_{\max} = \dfrac{k_c}{m\left|\omega_1^2-\omega_0^2\right|}$



The analytic result for the Q

$$Q = \frac{\omega_r}{\Delta\omega}$$

ω_r is the resonance frequency and $\Delta\omega$ is the bandwidth (full width when the output voltage is $V_{\max}/\sqrt{2}$)

$$
\begin{aligned}
\text{Quality Factor} = \Bigg\{ & \Bigg(3\,k\,m\,\tau^2 \sqrt{\Big(2\,m^2 + 2\,(k + kc\,kv)\,m\,\tau^2 - \gamma^2\,\tau^2 +} \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Bigg) \Big/ \Bigg(\sqrt{\Big(2\,m^6 + 6\,(k} \\
& - 2\,kc\,kv)\,m^5\,\tau^2 + 2\,\gamma^6\,\tau^6 - 2\,\gamma^4\,\tau^4 \\
& \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} + 4\,(k + kc\,kv)\,m\,\gamma^2 \\
& \tau^4\,\Big(-3\,\gamma^2\,\tau^2 + 2\,\sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) - \\
& m^4\,\Big(3\,\gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 - 6\,k^2\,\tau^4 + 6\,k\,kc\,kv\,\tau^4 - 15\,kc^2\,kv^2\,\tau^4 + 2 \\
& \quad \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) + m^2\,\tau^2\,\Big(-3\,\gamma^4 \\
& \quad \tau^2 - 18\,kc\,kv\,\gamma^3\,\tau^3 + 12\,kc\,kv\,\gamma\,\tau \\
& \quad \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} - 2\,(k + kc\,kv)^2 \\
& \quad \tau^2\,\sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)} + \gamma^2\,\Big(15\,k^2\,\tau^4 \\
& \quad\quad + 30\,k\,kc\,kv\,\tau^4 + 15\,kc^2\,kv^2\,\tau^4 + 2 \\
& \quad\quad \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Big) + 2\,m^3\,\tau^2\,\Big(\\
& \quad k^3\,\tau^4 + 3\,k^2\,kc\,kv\,\tau^4 + k\,\Big(6\,\gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 + 3\,kc^2\,kv^2\,\tau^4 - 2 \\
& \quad\quad \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big) + kc\,kv\,\Big(-3 \\
& \quad\quad \gamma^2\,\tau^2 + 18\,kc\,kv\,\gamma\,\tau^3 + kc^2\,kv^2\,\tau^4 + 4 \\
& \quad\quad \sqrt{m^4 + 2\,(k - 2\,kc\,kv)\,m^3\,\tau^2 - 4\,(k + kc\,kv)\,m\,\gamma^2\,\tau^4 + \gamma^4\,\tau^4 + m^2\,\tau^2\,\left(-\gamma^2 - 6\,kc\,kv\,\gamma\,\tau + (k + kc\,kv)^2\,\tau^2\right)}\Big)\Big)\Big)}\Bigg)\Bigg\}
\end{aligned}
$$

Description of the resonator design

The resonator design is a square shaped block of single crystal silicon with dimensions of 320x320x28 um^3 (design H1). Its main resonance mode is the so called square extensional (SE) resonance, which is characterized by its zoom-in/zoom-out oscillation. The resonance is excited by a piezoelectric AlN thin film on top of the resonator block. The electrically conductive (p-doped) silicon block acts as the bottom electrode, and a molybdenum thin film has been patterned to provide the top electrode. See reference [1] for a general description of the SE resonator. Reference [2] discusses piezoelectric excitation of the SE resonance mode.

Figure 3 shows how the resonator is recommended to be connected.

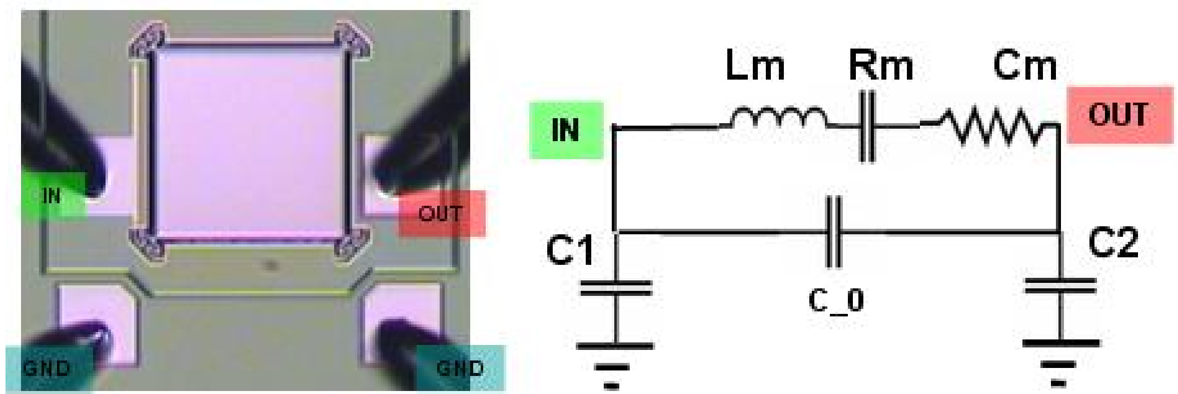
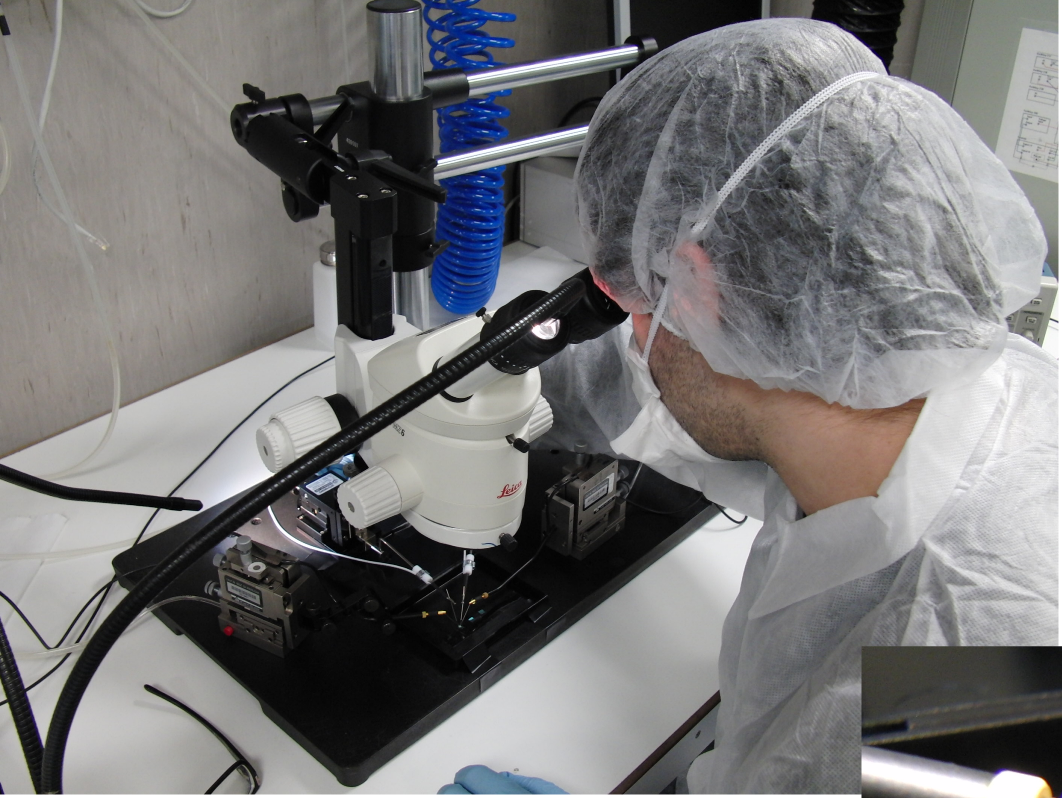
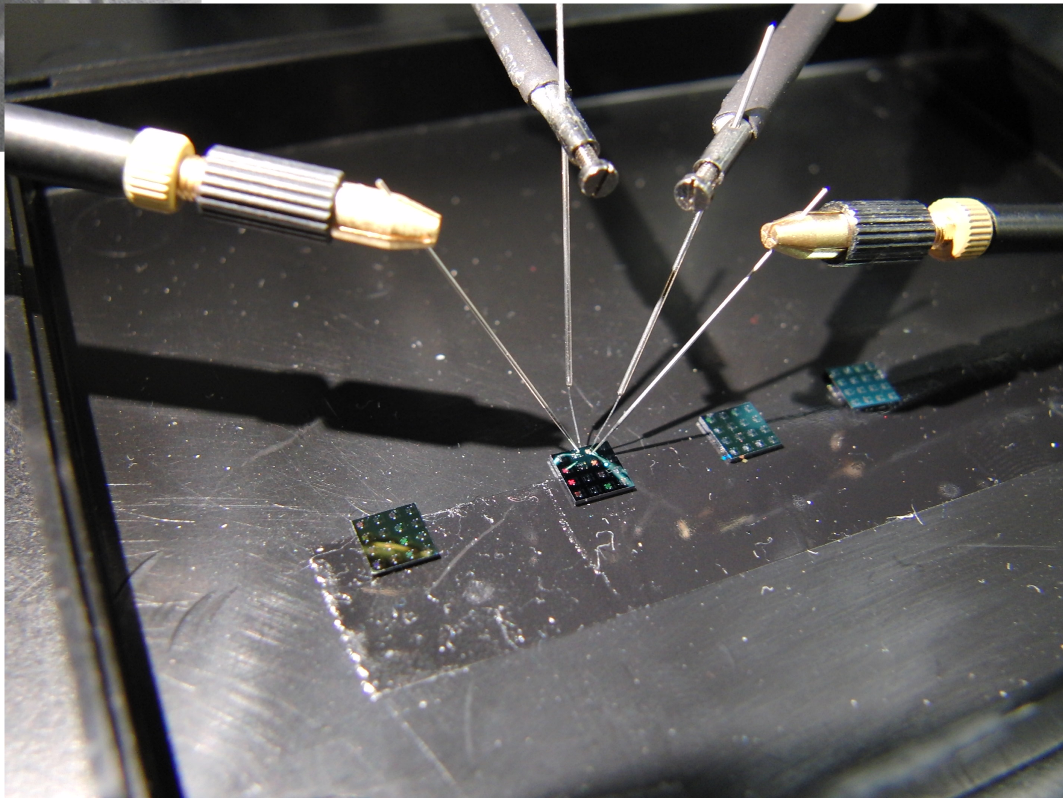


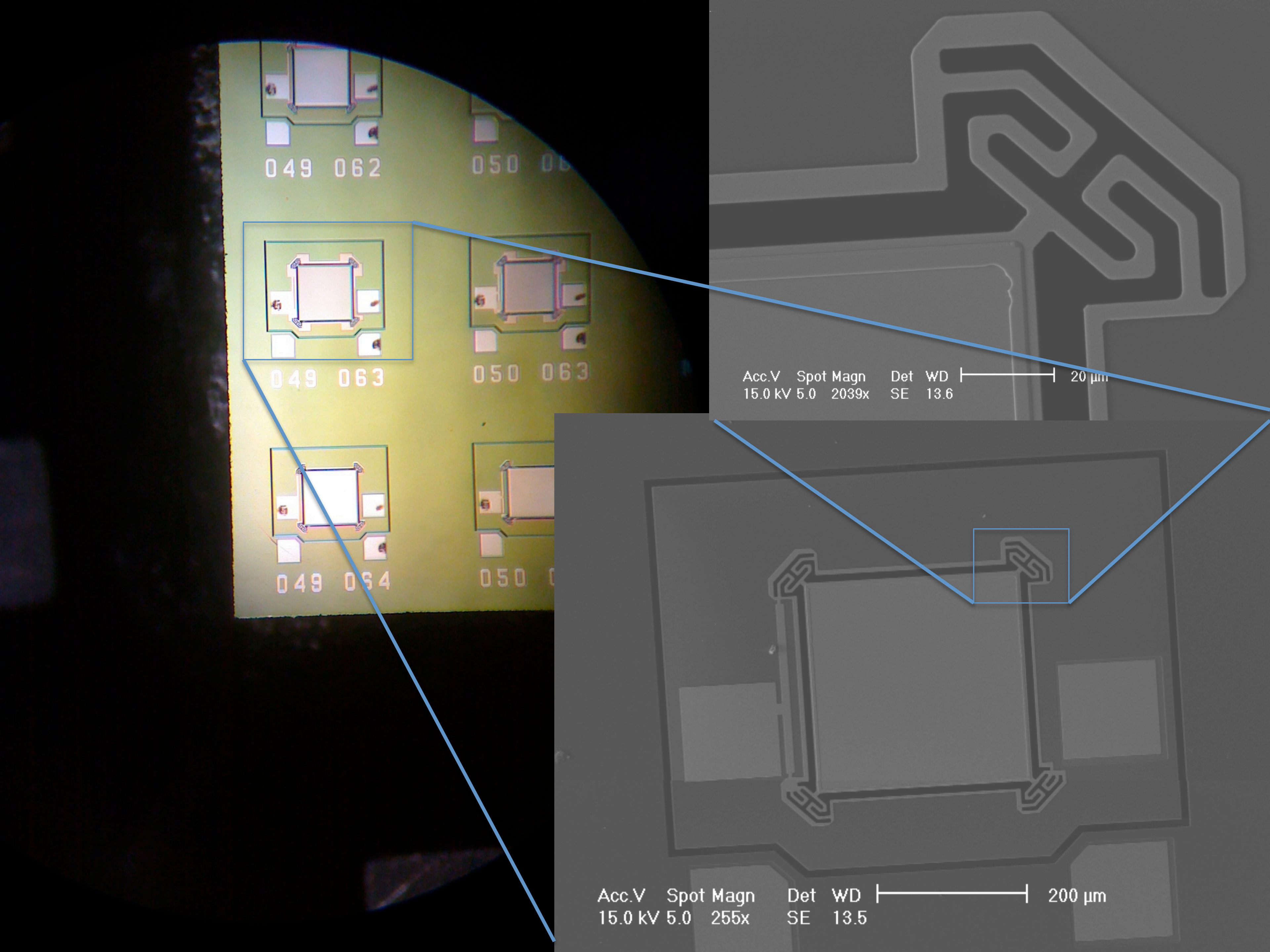
Figure 3: Electrical connection of the resonator.

References
[1] V. Kaajakari et al., "Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications," *IEEE Electron Device Letters* 25, no. 4 (4, 2004): 173-175.
[2] A. Jaakkola et al., "Piezoelectrically transduced Single-Crystal-Silicon Plate Resonators," in *IEEE Ultrasonics Symposium* (presented at the IEEE Ultrasonics Symposium, Beijing, China, 2008), 2181 – 2184



Probing station




049 062
050
049 063
050 063
049 064
050
Acc.V Spot Magn Det WD 20 µm
15.0 kV 5.0 2039x SE 13.6
Acc.V Spot Magn Det WD 200 µm
15.0 kV 5.0 255x SE 13.5

The measurements

Resonators design

Four separate chips have been provided, each chip contains 16 resonators.

Each of the 16 resonators has a different design, their size is varied (so the resonance frequency)

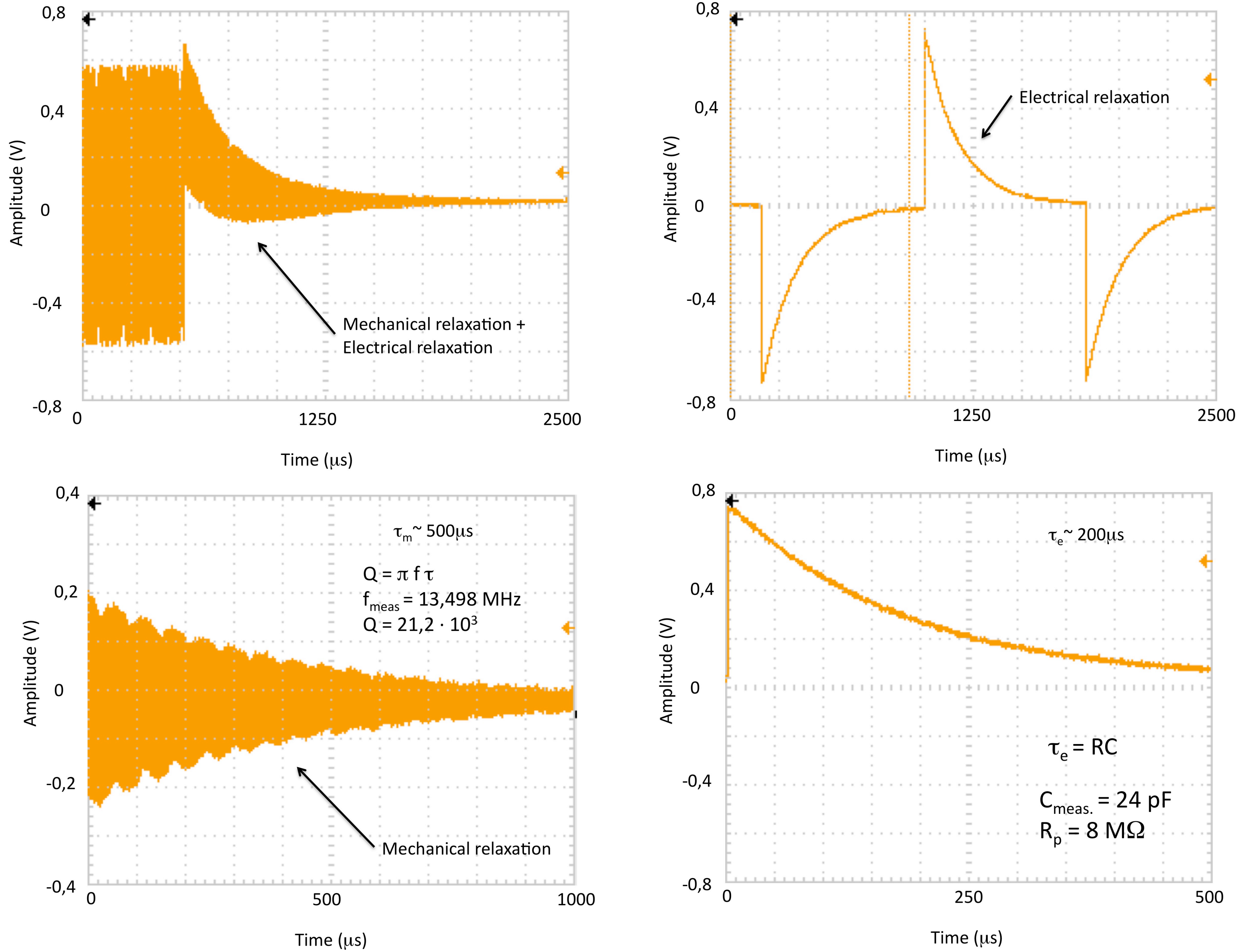
resonator lateral size (um)

311	287	263	239
314	290	266	242
317	293	269	245
320	296	272	248

main mode frquency (MHz)

13.9	15.1	16.4	18.1
13.8	14.9	16.2	17.9
13.6	14.7	16.1	17.6
13.5	14.6	15.9	17.4

VTT Membrane characterization



Electrical parameters (chip 2)

Capacity (pF)

26.52±0.07	24.00±0.06	21.26±0.05	18.73±0.05
29.46±0.06	25.84±0.06	21.83±0.05	21.26±0.05
27.62±0.06	24.48±0.06	21.83±0.05	19.45±0.05
30.40±0.08	26.70±0.07	23.36±0.06	19.57±0.05

τ_e (μs)

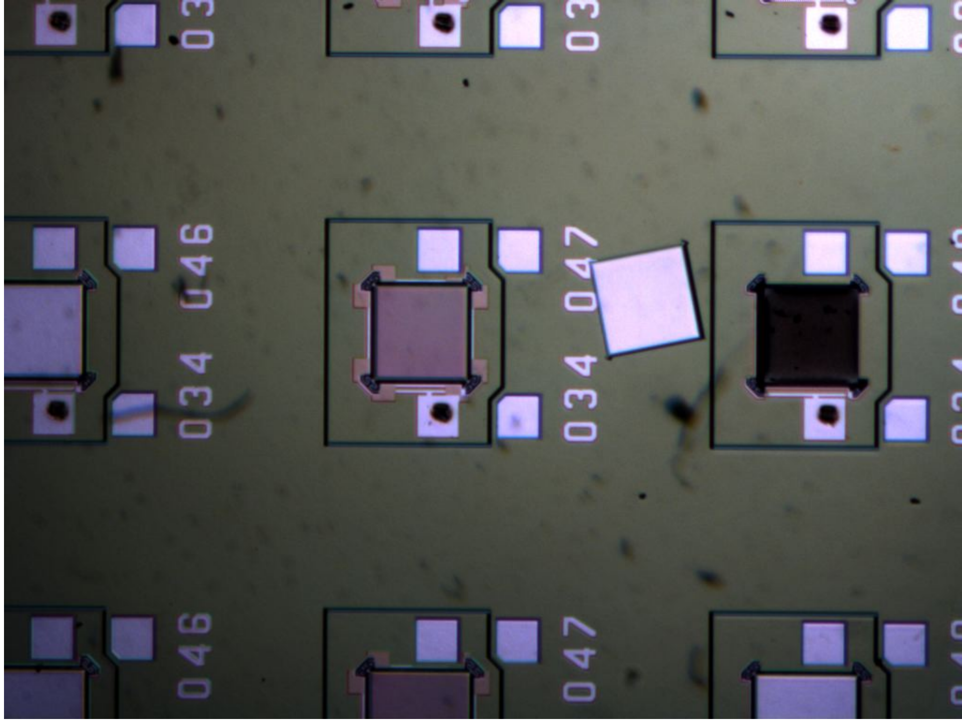
222±8	219±5	220±6	214±6
216±4	220±9	215±7	213±5
214±5	217±7	215±5	220±3
225±5	n.a.	209±3	217±3

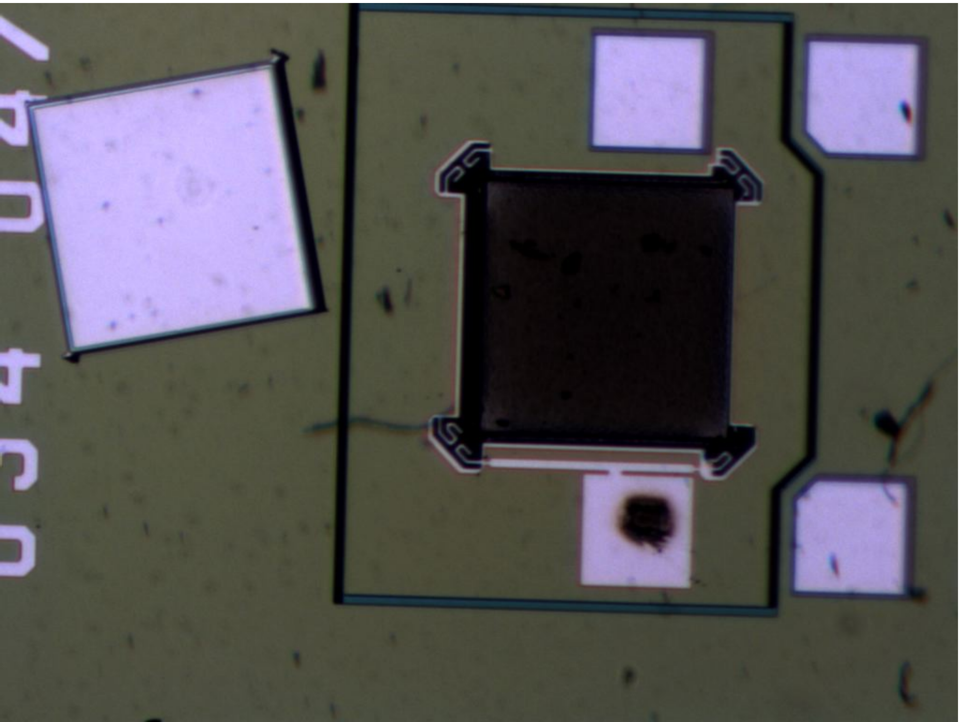
Resistance (MΩ)

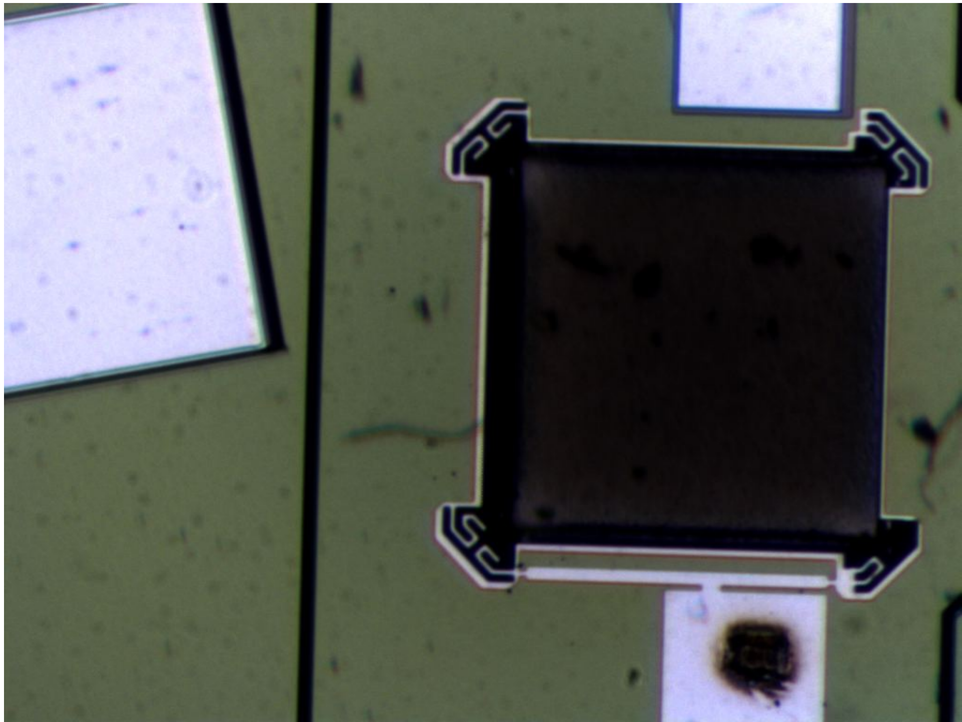
8.4±0.4	9.1±0.3	10.3±0.3	11.5±0.4
7.3±0.1	8.5±0.4	9.8±0.3	10.0±0.3
7.8±0.2	8.9±0.3	9.9±0.2	11.3±0.2
7.4±0.2	n.a.	8.9±0.2	11.1±0.2

Why?

The membrane was broken with an excitation of 4 volts at the resonance frequency







Mechanical parameters

τ_m (μs)

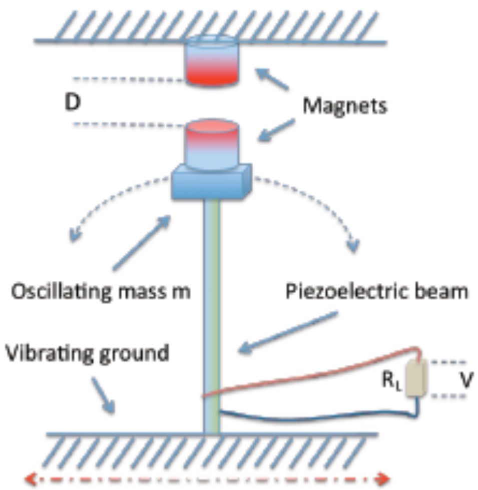
165±8	99±4	118±7	149±19
279±24	73±4	144±5	125±11
137±4	113±3	150±9	158±16
204±7	n.a.	146±8	158±10

Quality factors (declared 9000)

7200±350	4700±200	6080±360	8300±1000
12000±1000	3400±160	7300±260	6950±600
5860±160	5250±150	7580±480	8700±860
8700±300	n.a.	7300±380	8600±560

The vibration harvester 2.0

- ✓ Capable of harvesting energy on a broad-band ------------> Non-resonant system
- ✓ No need for frequency tuning ------------> “Transfer function” with wide frequency resp.
- ✓ Capable of harvesting energy at low frequency ------------> Low frequency operated



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Nonlinear Energy Harvesting

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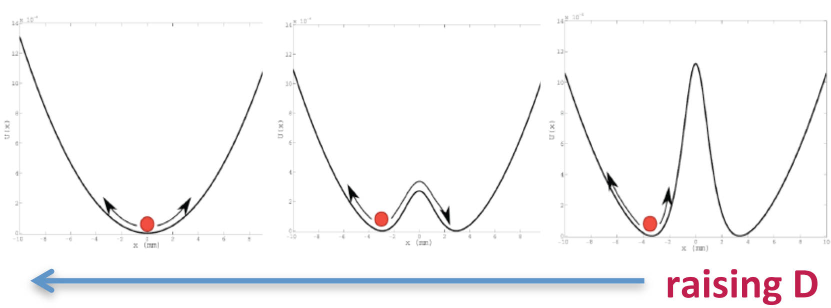
Ambient energy harvesting has been in recent years the recurring object of a number of research efforts aimed at providing an autonomous solution to the powering of small-scale electronic mobile devices. Among the different solutions, vibration energy harvesting has played a major role due to the almost universal presence of mechanical vibrations. Here we propose a new method based on the exploitation of the dynamical features of stochastic nonlinear oscillators. Such a method is shown to outperform standard linear oscillators and to overcome some of the most severe limitations of present approaches. We demonstrate the superior performances of this method by applying it to piezoelectric energy harvesting from ambient vibration.

DOI: 10.1103/PhysRevLett.102.080601 PACS numbers: 05.40.Ca, 05.10.Ln, 05.45.−a, 84.60.−h



Result: output power is maximum for an optimal nonlinear regime

Let's look at an example of non-linear oscillator:

the Duffing Oscillator

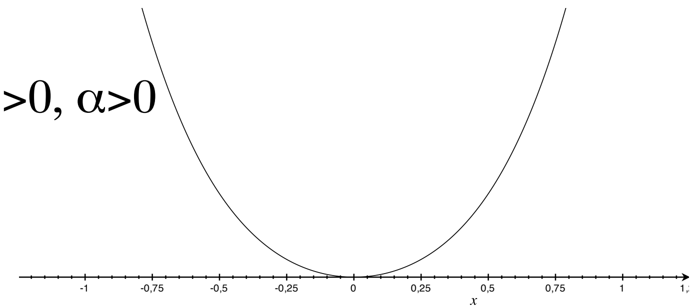
$$\ddot{x} + \delta\dot{x} + \beta x + \alpha x^3 = \gamma\cos\omega t$$

$$U(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4$$

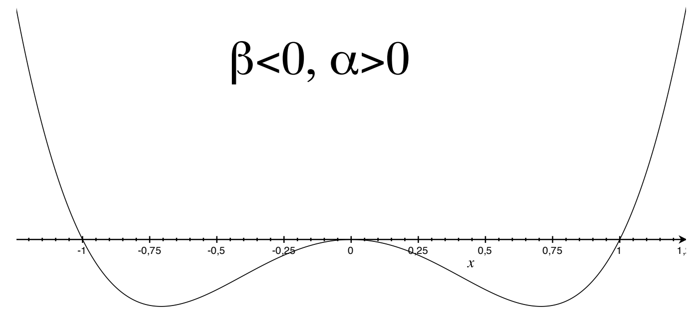
The Duffing Potential

$$U(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4$$

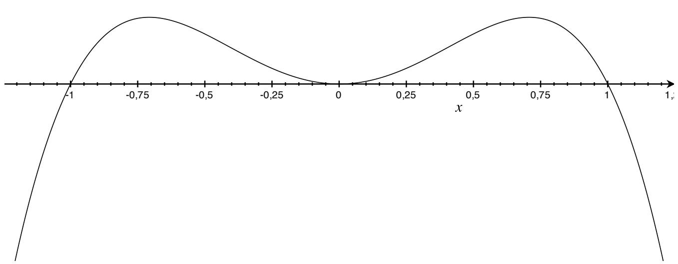
$\beta>0,\ \alpha>0$



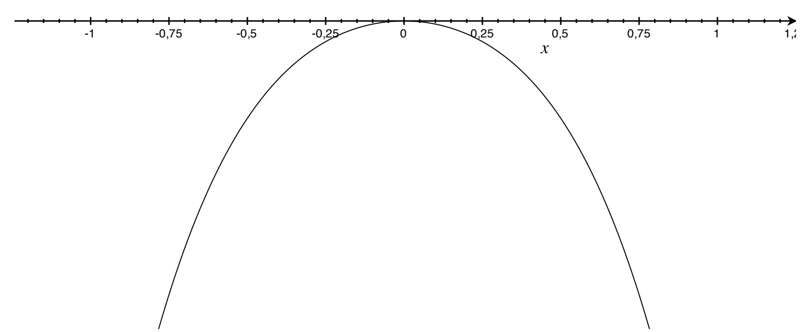
$\beta<0,\ \alpha>0$



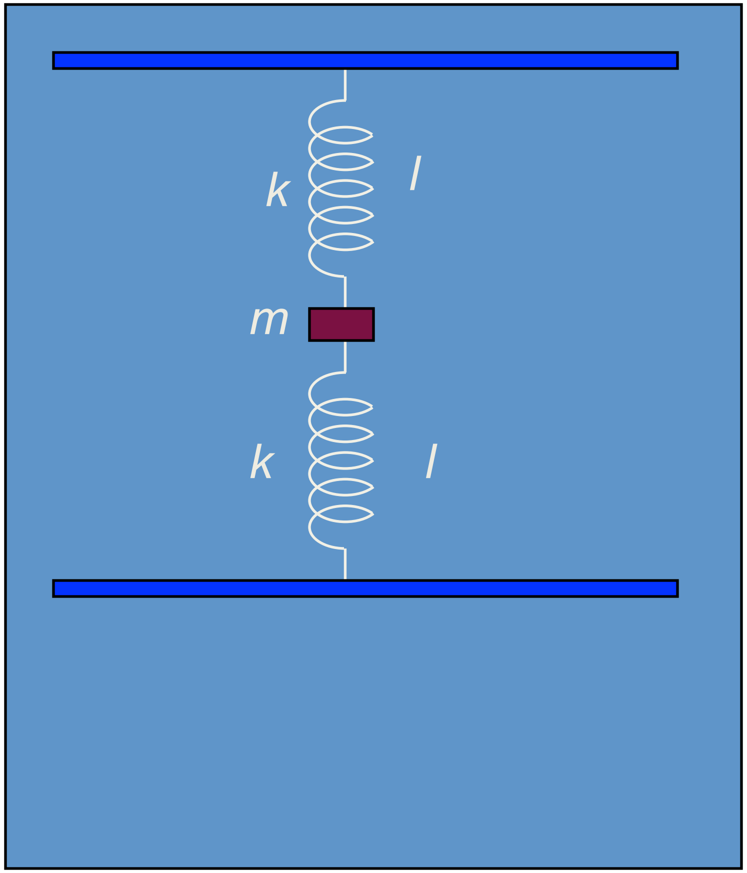
$\beta>0,\ \alpha<0$



$\beta<0,\ \alpha<0$

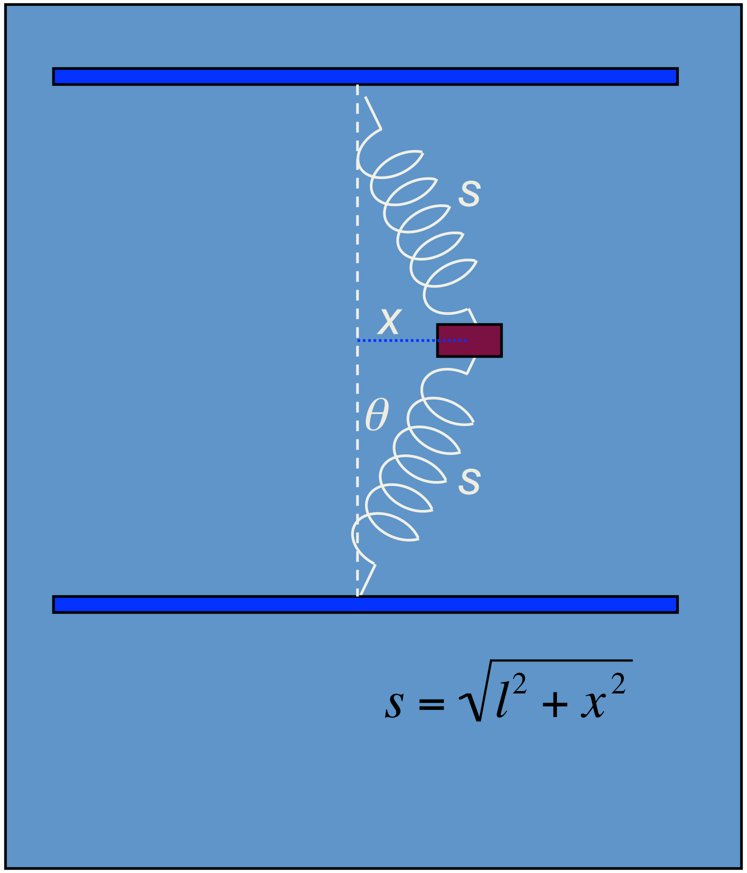


A two springs system



- A mass is held between two springs.
  - Spring constant *k*
  - Natural length *l*
- Springs are on a horizontal surface.
  - Frictionless
  - No gravity

Transverse Displacement



- The force for a displacement is due to both springs.
  - Only transverse component
  - Looks like its harmonic

$$F = -2k\left(\sqrt{l^2 + x^2} - l\right)\sin\theta$$

$$= -2k\left(\sqrt{l^2 + x^2} - l\right)\frac{x}{\sqrt{l^2 + x^2}}$$

$$= -2kx\left(1 - \frac{1}{\sqrt{1 + x^2/l^2}}\right)$$

Purely Nonlinear

- The force can be expanded as a power series near equilibrium.
  - Expand in *x/l*

$$F = -2kl\frac{x}{l}\left(1 - \frac{1}{\sqrt{1 + x^2/l^2}}\right)$$

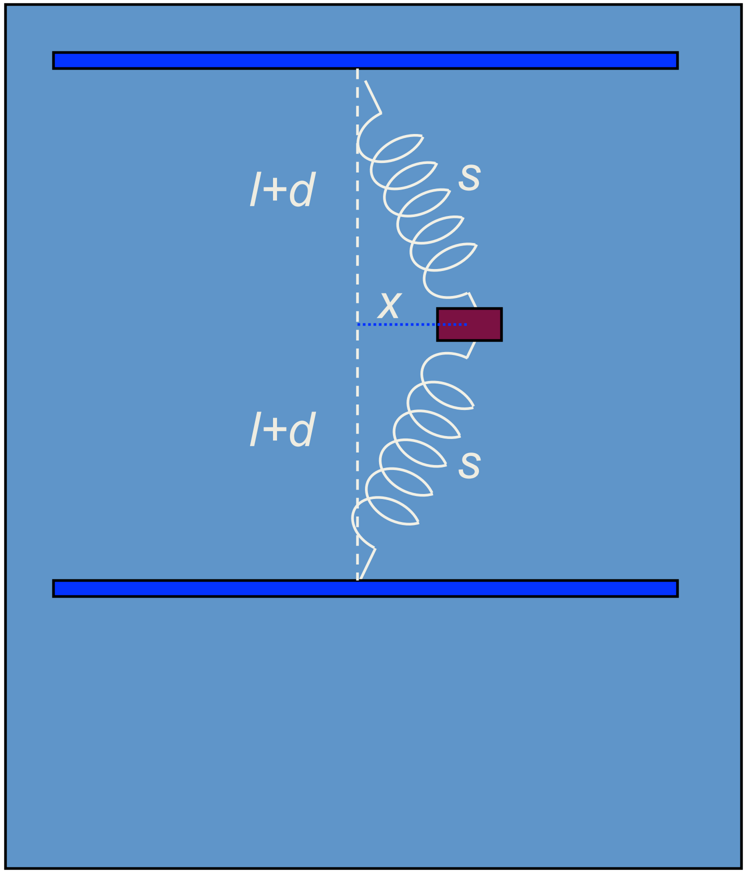
- The lowest order term is non-linear.

$$F \cong -kl\left(\frac{x}{l}\right)^3 + \dots$$

- Quartic potential
  - Not just a perturbation

$$V \cong \frac{k}{4l^2}x^4 + \dots$$

Mixed Potential



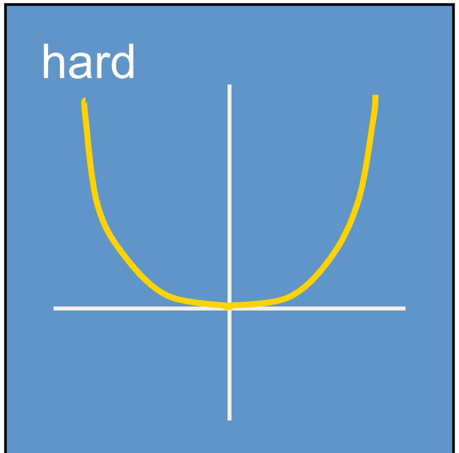
- Typical springs are not at natural length.
  - Approximation includes a linear term

$$F \cong -\frac{2kd}{l}x - \frac{k(l-d)}{l^3}x^3 + \ldots$$

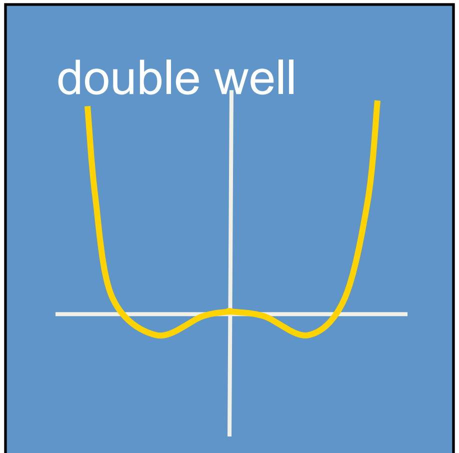
$$V \cong \frac{kd}{l}x^2 + \frac{k(l-d)}{4l^3}x^4 + \ldots$$

Quartic Potentials

- The sign of the forces influence the shape of the potential.



$$V = \frac{k}{2}x^2 + \frac{k\alpha}{4}x^4$$



$$V = -\frac{k}{2}x^2 + \frac{k\alpha}{4}x^4$$

Driven System

- Assume a more complete, realistic system.
  - Damping term
  - Driving force

$$m\ddot{x} = -\beta\dot{x} - kx - k\alpha x^3 + f\cos\omega t$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x + \alpha\omega_0^2 x^3 = f\cos\omega t$$

- Rescale the problem:
  - Set t such that $\omega_0^2 = k/m = 1$
  - Set x such that $k\alpha/m = 1$

- This is the Duffing equation

$$\ddot{x} + \gamma\dot{x} + x + x^3 = f\cos\omega t$$

Steady State Solution

- Try a solution, match terms

$$x(t) = A(\omega)\cos[\omega t - \theta(\omega)]$$

$$\ddot{x} + \gamma\dot{x} + x + x^3 = f\cos\omega t$$

$$A(1-\omega^2)\cos(\omega t - \theta) - A\gamma\omega\sin(\omega t - \theta) + A^3\cos^3(\omega t - \theta) = f\cos\omega t$$

trigonometric identities

$$\cos^3(\omega t - \theta) = \tfrac{3}{4}\cos(\omega t - \theta) + \tfrac{1}{4}\cos 3(\omega t - \theta)$$

$$f\cos\omega t = f\cos\theta\cos(\omega t - \theta) - f\sin\theta\sin(\omega t - \theta)$$

$$\begin{aligned}
&[A(1-\omega^2 + \tfrac{3}{4}A^2) - f\cos\omega t]\cos(\omega t - \theta) \\
&+[-A\gamma\omega + f\sin\omega t]\sin(\omega t - \theta) \\
&+\tfrac{1}{4}A^3\cos 3(\omega t - \theta) \\
&= 0
\end{aligned}$$

$$f\cos\omega t = A(1-\omega^2 + \tfrac{3}{4}A^2)$$

$$f\sin\omega t = A\gamma\omega$$

$$\tfrac{1}{4}A^3\cos 3(\omega t - \theta) \approx 0$$

Amplitude Dependence

- Find the amplitude-frequency relationship.
  - Reduces to forced harmonic oscillator for A → 0

$$A = \frac{f}{\sqrt{[(1-\omega^2)^2 + (\gamma\omega)^2]}}$$

$$f^2 \cos^2 \omega t = A^2(1-\omega^2 + \tfrac{3}{4}A^2)^2$$

$$f^2 \sin^2 \omega t = A^2\gamma^2\omega^2$$

$$f^2 = A^2[(1-\omega^2 + \tfrac{3}{4}A^2)^2 + \gamma^2\omega^2]$$

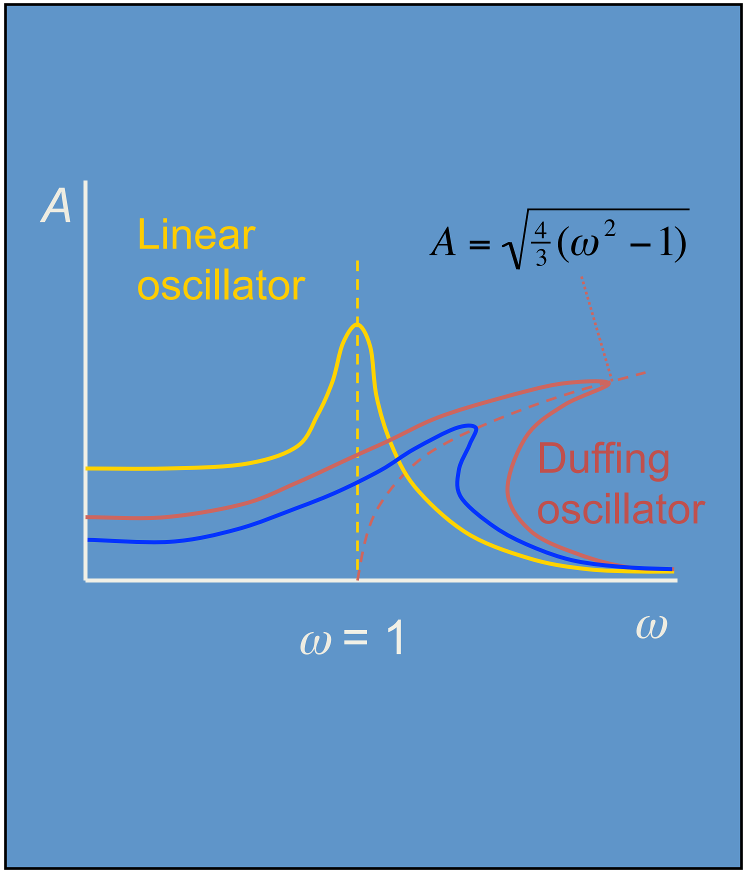
- Find the case for minimal damping and driving force.
  - *f*, γ both near zero
  - Defines resonance condition

$$0 = A^2[(1-\omega^2 + \tfrac{3}{4}A^2)^2 + 0]$$

$$0 = 1-\omega^2 + \tfrac{3}{4}A^2$$

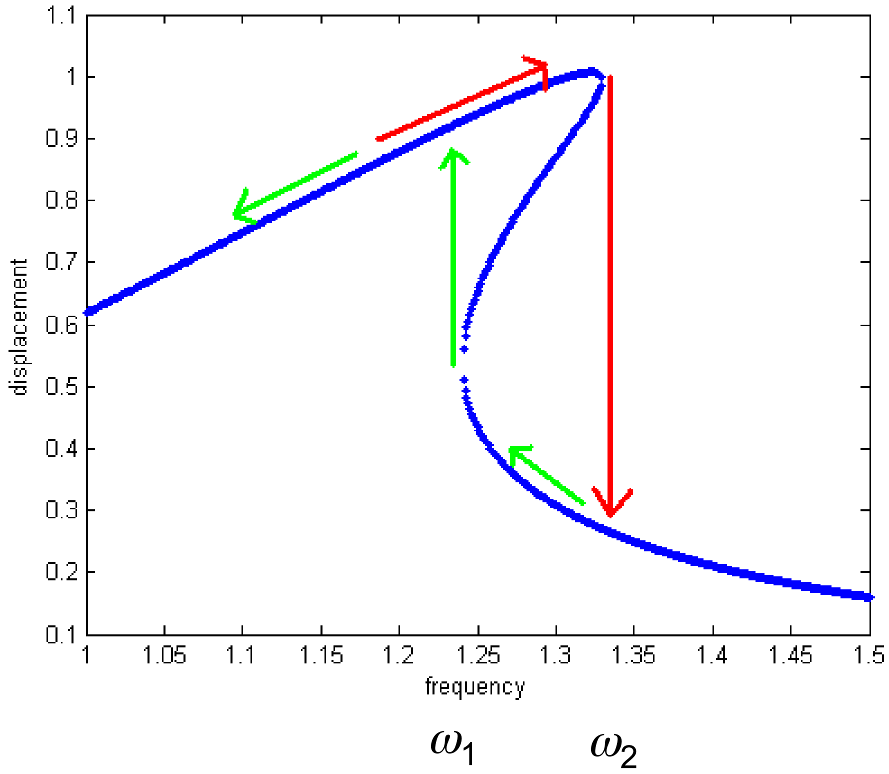
$$A(\omega) = \sqrt{\tfrac{4}{3}(\omega^2 - 1)}$$

Nonlinear Resonance Frequency



- The resonance frequency of a linear oscillator is independent of amplitude.
- The resonance frequency of a Duffing oscillator increases with amplitude.

… brings to hysteresis



- A Duffing oscillator behaves differently for increasing and decreasing frequencies.
  - Increasing frequency has a jump in amplitude at ω_2
  - Decreasing frequency has a jump in amplitude at ω_1

- This is hysteresis.

Nonlinear Resonance

(in general…)

Nonlinear resonance seems not to be so much different from the (linear) resonance of a harmonic oscillator. But both, the dependency of the eigenfrequency of a nonlinear oscillator on the amplitude and the nonharmoniticity of the oscillation lead to a behavior that is impossible in harmonic oscillators, namely: the foldover effect and superharmonic resonance.

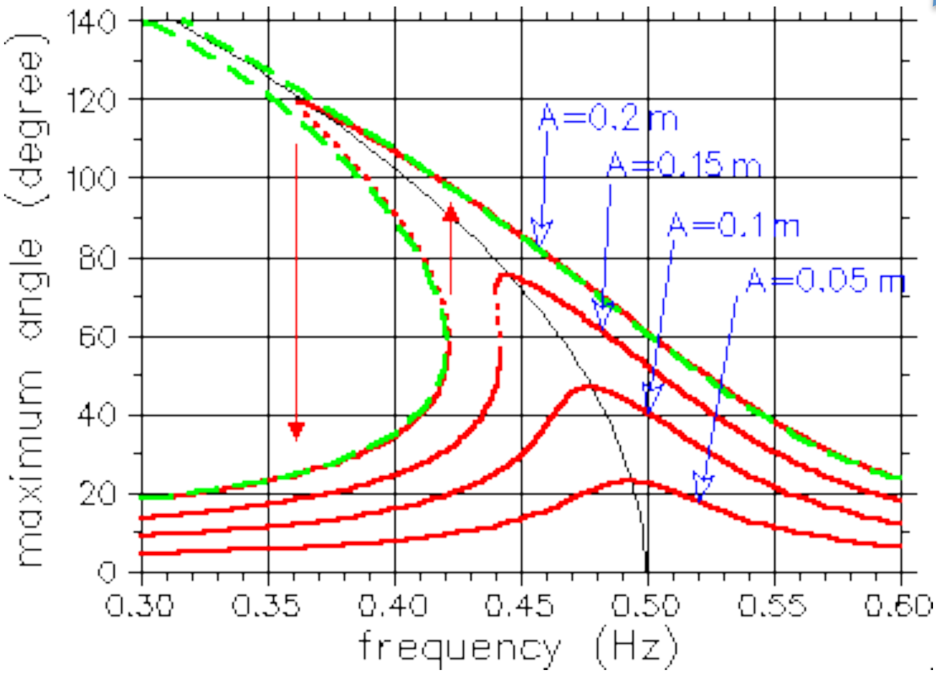
Both effects are especially important in the case of weak damping.

The foldover effect

The foldover effect got its name from the bending of the resonance peak in a amplitude versus frequency plot. This bending is due to the frequency-amplitude relation which is typical for nonlinear oscillators.

Foldover effect for a pendulum

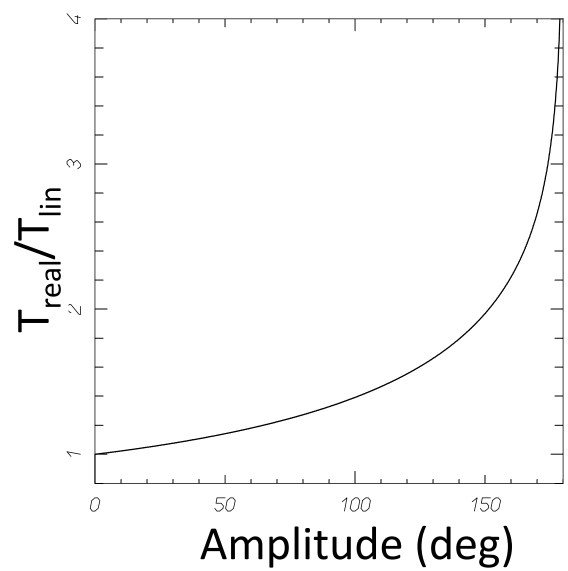
$$g = 9.81 m/\sec^2, l = 1 m, \gamma = 0.4 \sec^{-1}$$



The pendulum eq.:

$$\ddot{\varphi} = -\gamma\dot{\varphi} - \omega_0^2 \sin\varphi + f\cos\omega t$$

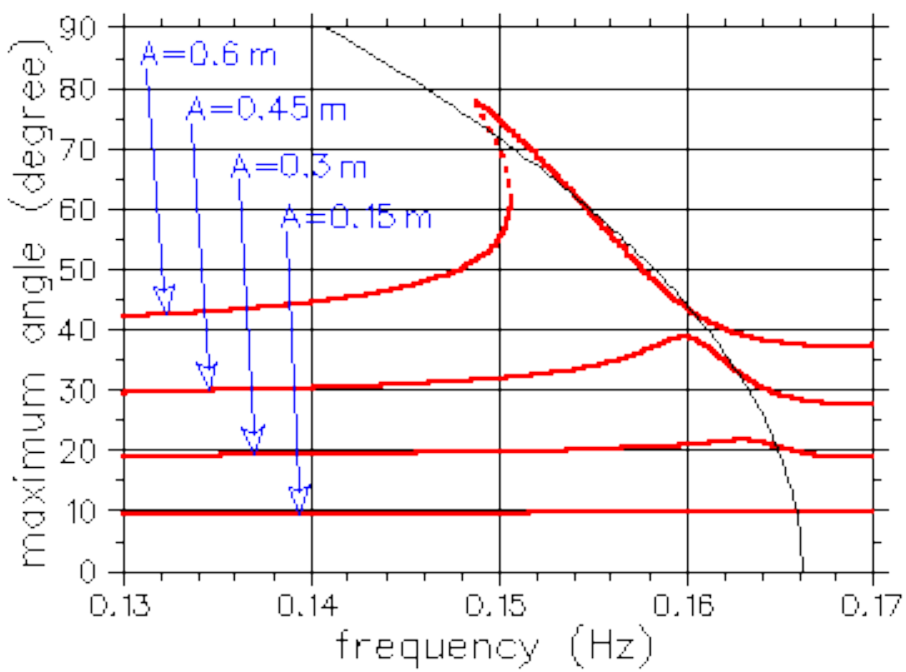
$$\omega_0^2 = \frac{g}{l}$$



The superharmonic resonance

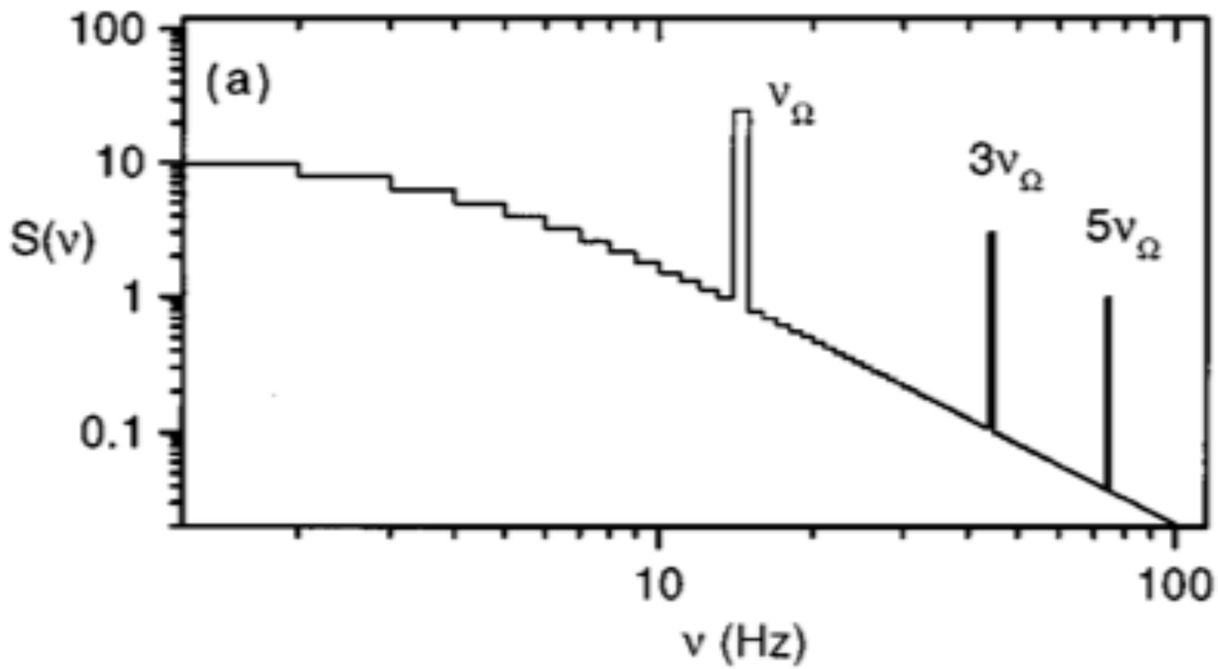
Nonlinear oscillators do not oscillate sinusoidal.
Superharmonic resonance is simply the resonance with one of this higher harmonics of a nonlinear oscillation. In an amplitude/frequency plot appear additional resonance peaks. In general, they appear at driving frequencies which are integer fractions of the fundamental frequency.

$$g = 9.81 m/\sec^2, l = 1m, \gamma = 0.1\sec^{-1}$$



Bistable Duffing

In case of a bistable oscillator the frequency response for an overdamped system is highly spread in the low frequency region.

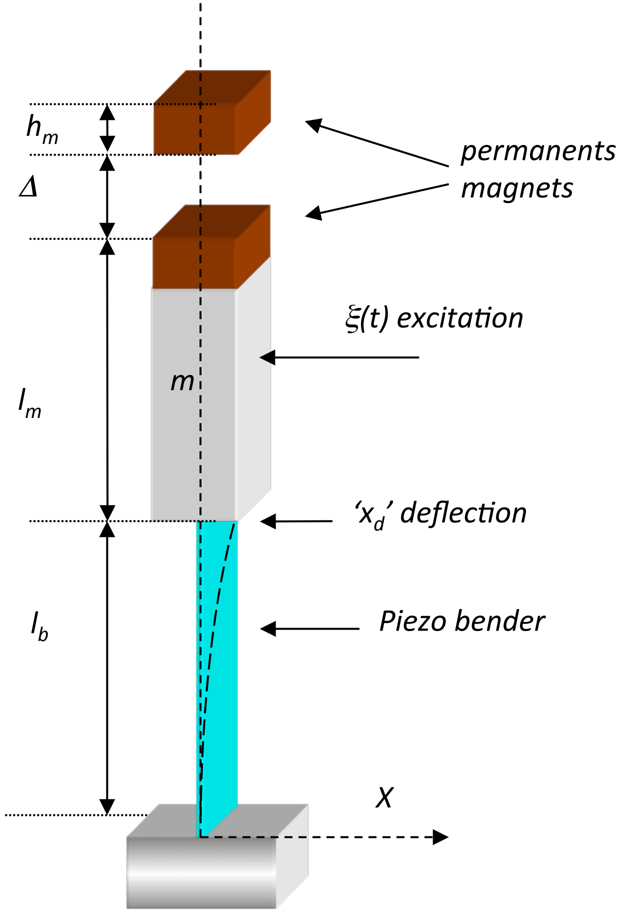


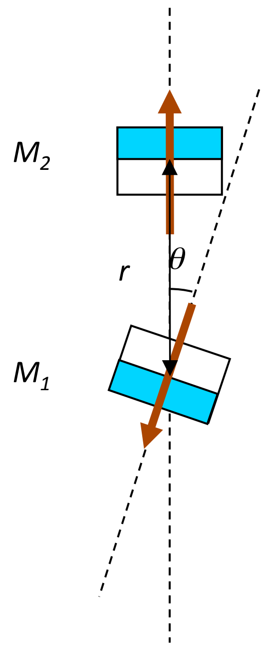
Gammaitoni et al. Reviews of Modern Physics 1998

Noise energy harvesting

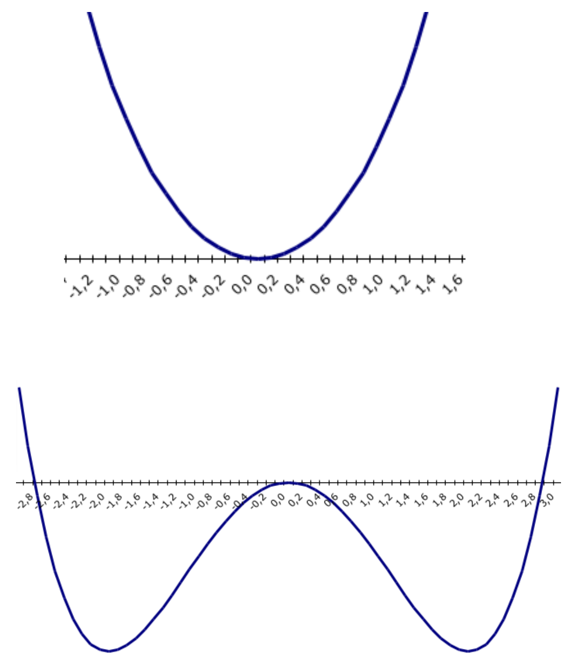
NON-Linear mechanical oscillators

NON-Linear Inverted pendulum



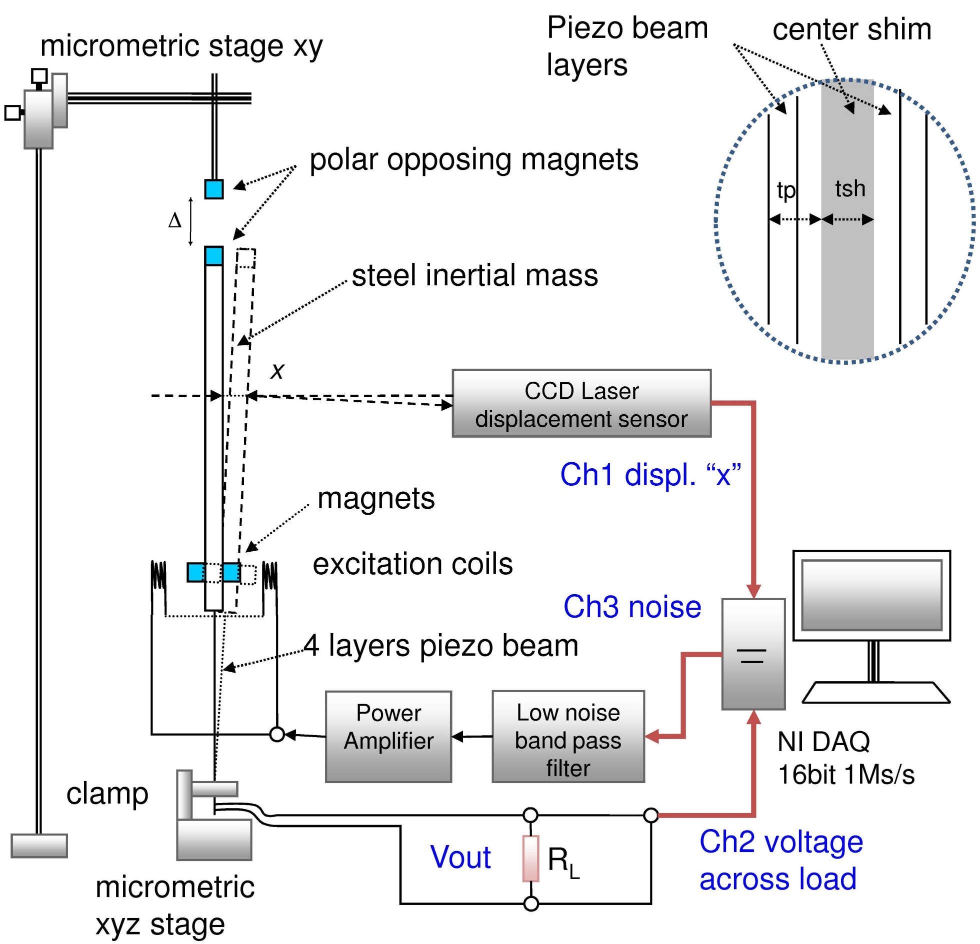


b)



Noise energy harvesting

NON-Linear mechanical oscillators



$$m\ddot{x} = -\frac{dU(x)}{dx} - \gamma\dot{x} - K_V V + \zeta_z$$

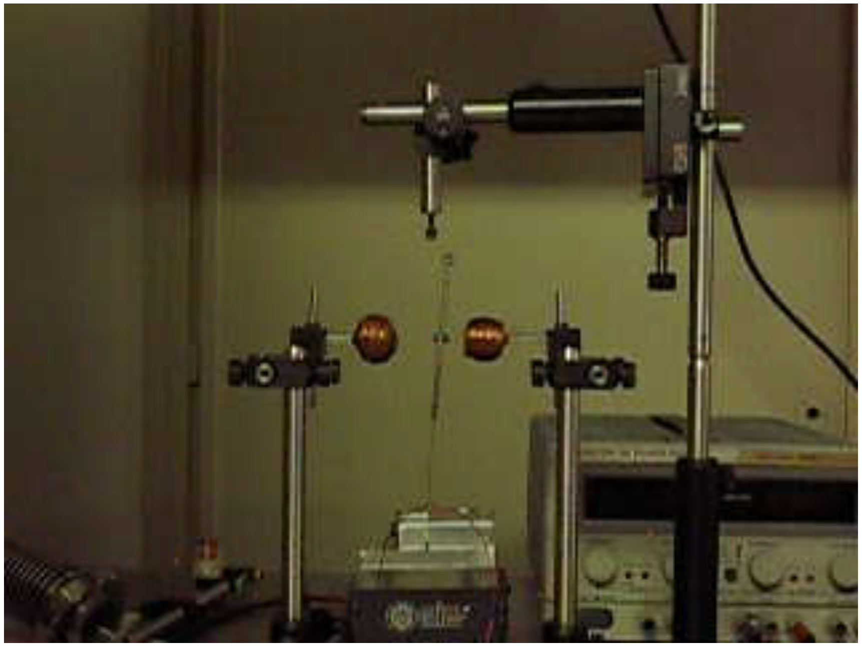
$$\dot{V} = K_c\dot{x} - \frac{1}{\tau_p}V$$

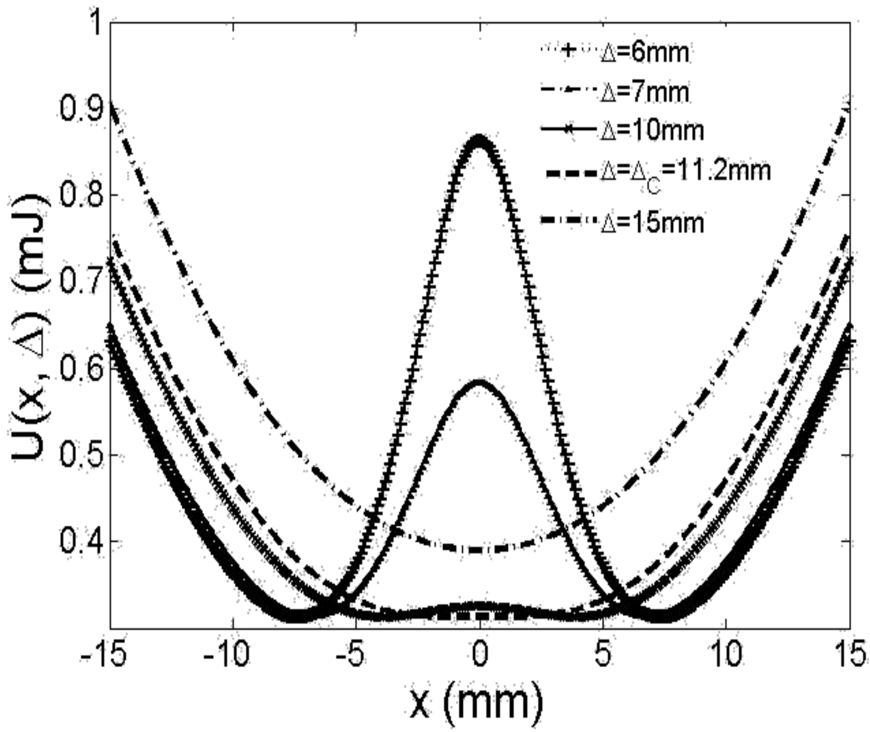
with

$$U(x) = kx^2 + (ax^2 + b\Delta^2)^{-3/2}$$

Noise energy harvesting

NON-Linear mechanical oscillators



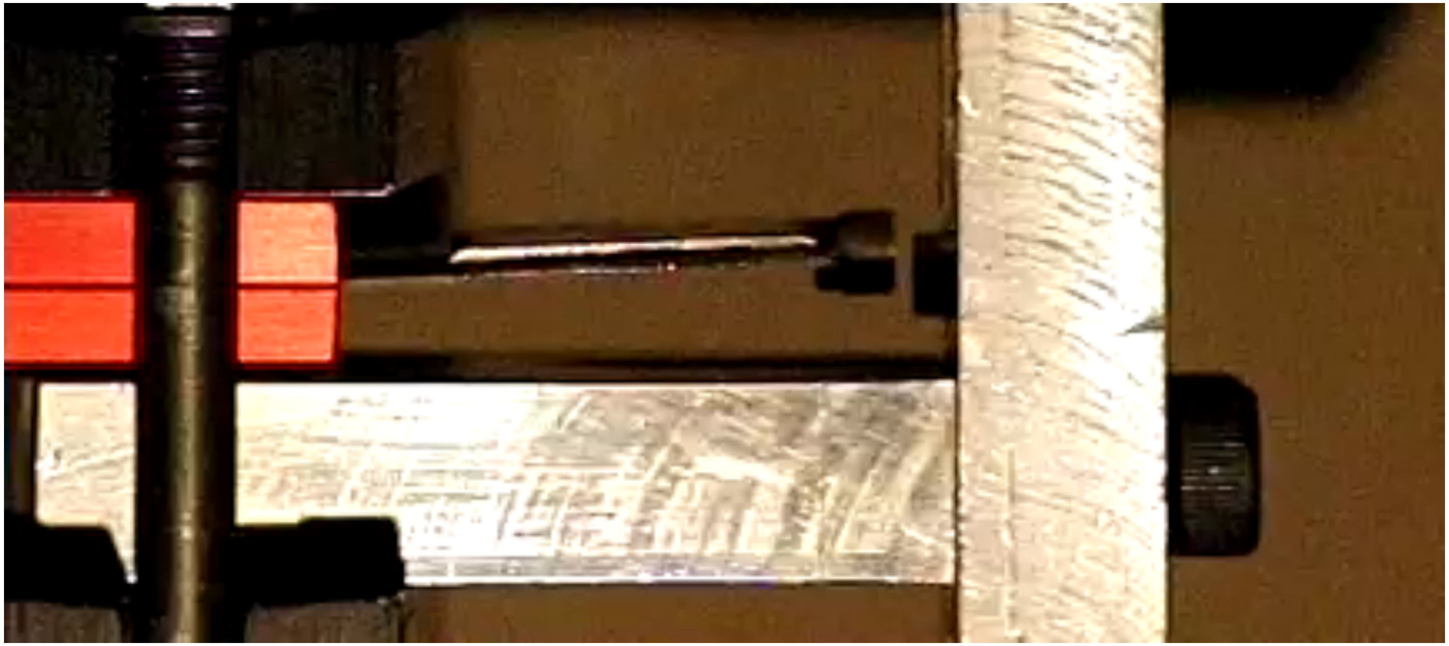


http://www.nipslab.org/node/1676

Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni
Physical Review Letters, 102, 080601 (2009)

Noise energy harvesting

NON-Linear mechanical oscillators



Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni
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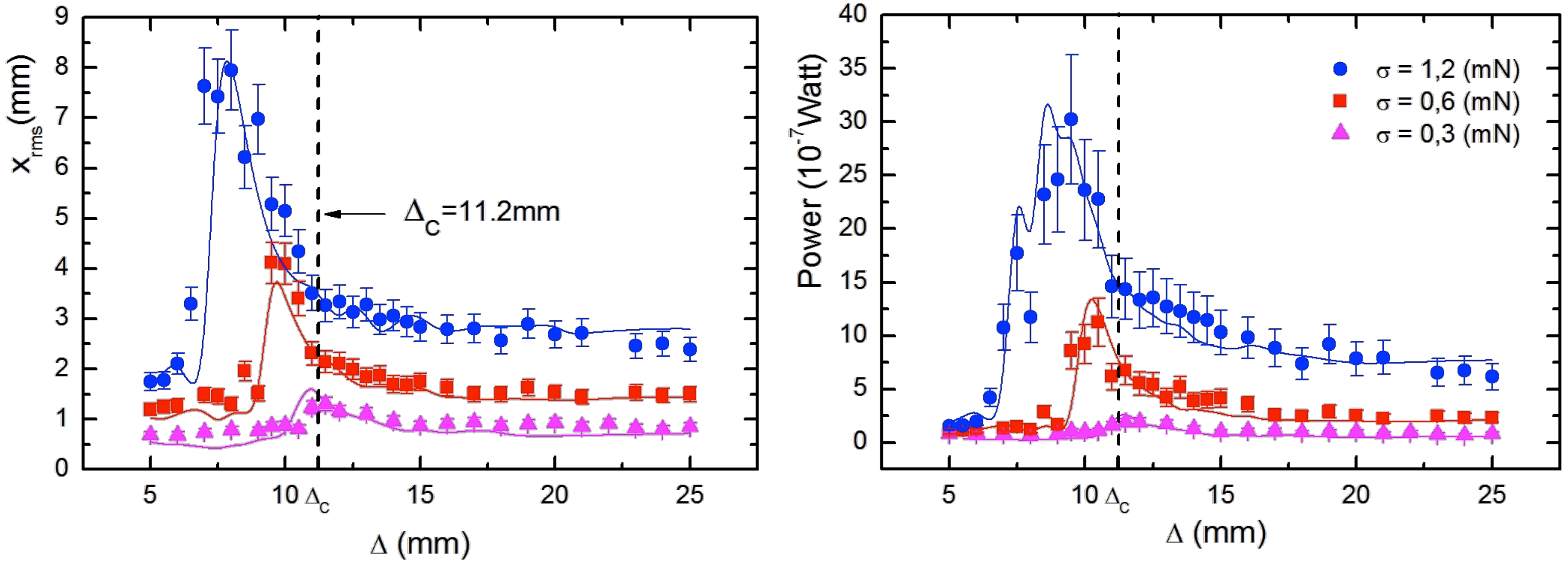
Noise energy harvesting

NON-Linear mechanical oscillators



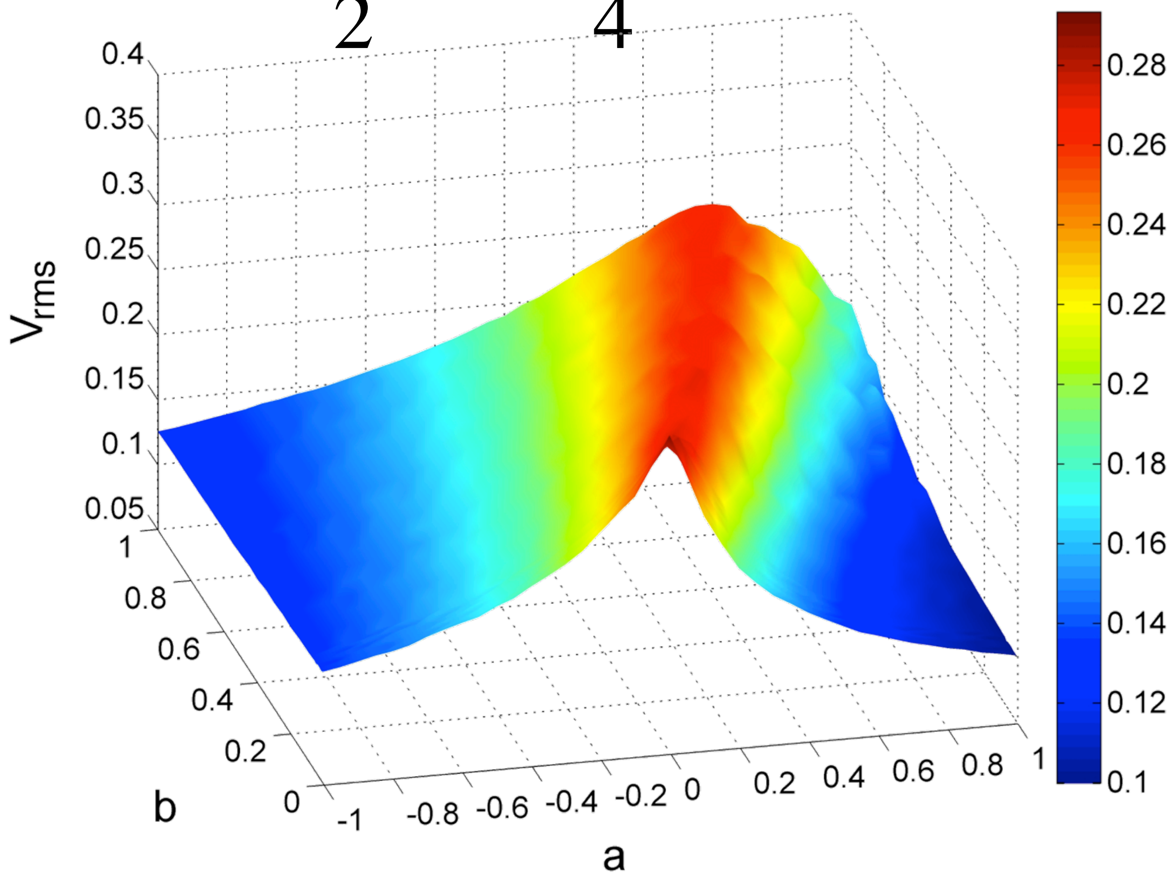
Nonlinear Energy Harvesting, F. Cottone; H. Vocca; L. Gammaitoni , Physical Review Letters, 102, 080601 (2009)

Noise energy harvesting

Non-linear systems

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$

Duffing potential

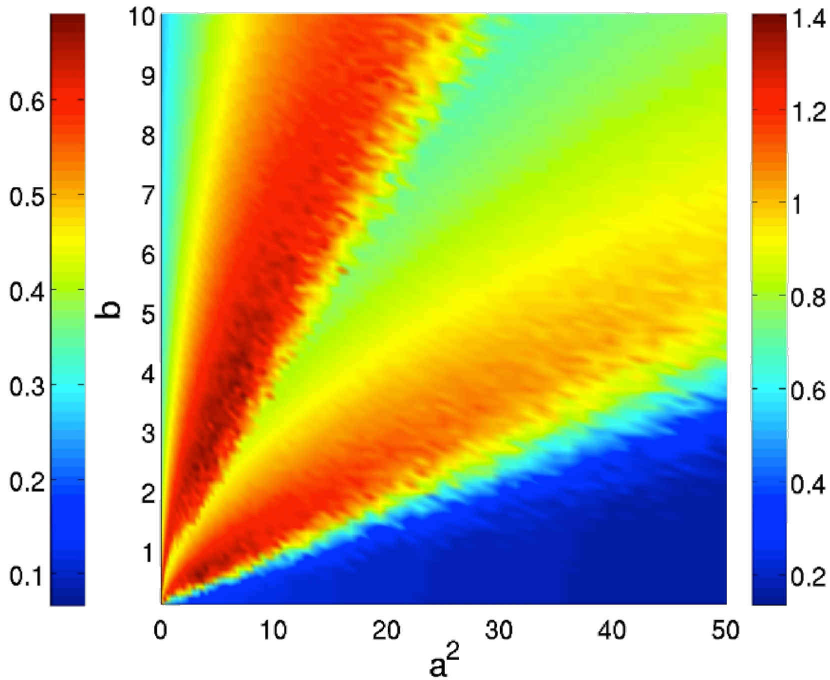


L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

Noise energy harvesting

Non-linear systems

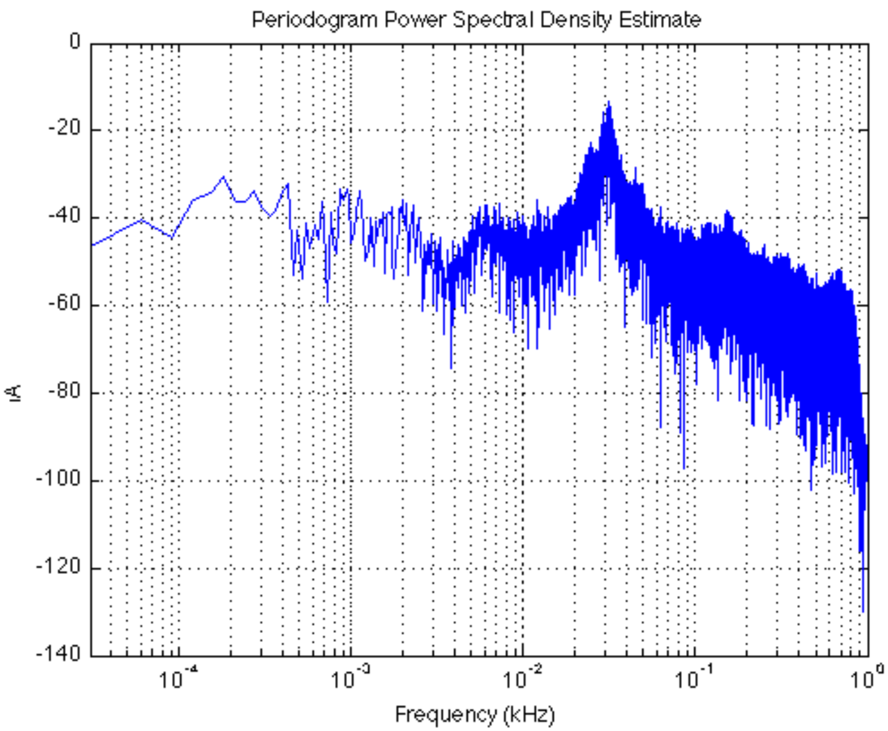
$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$$ Duffing potential

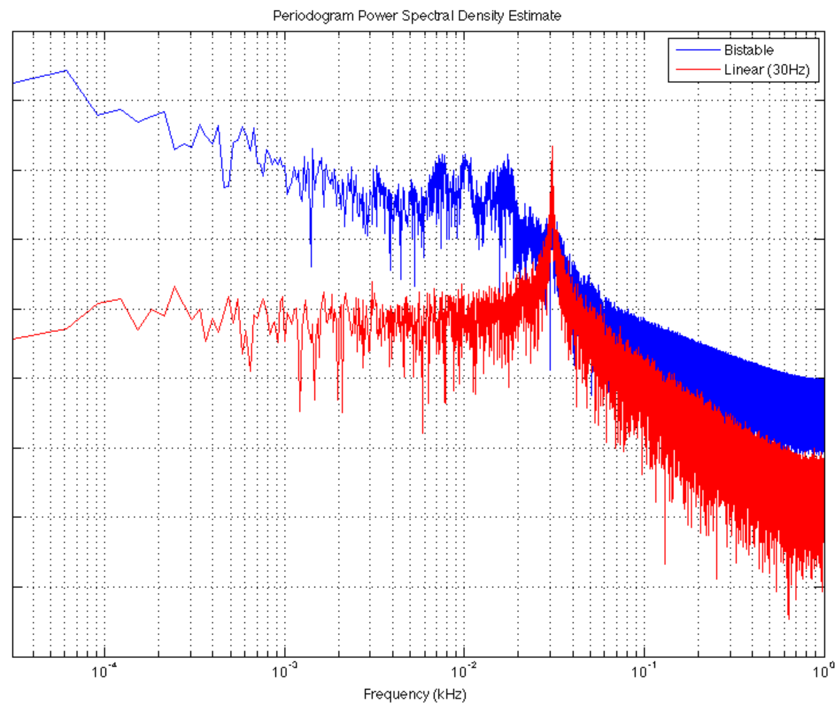


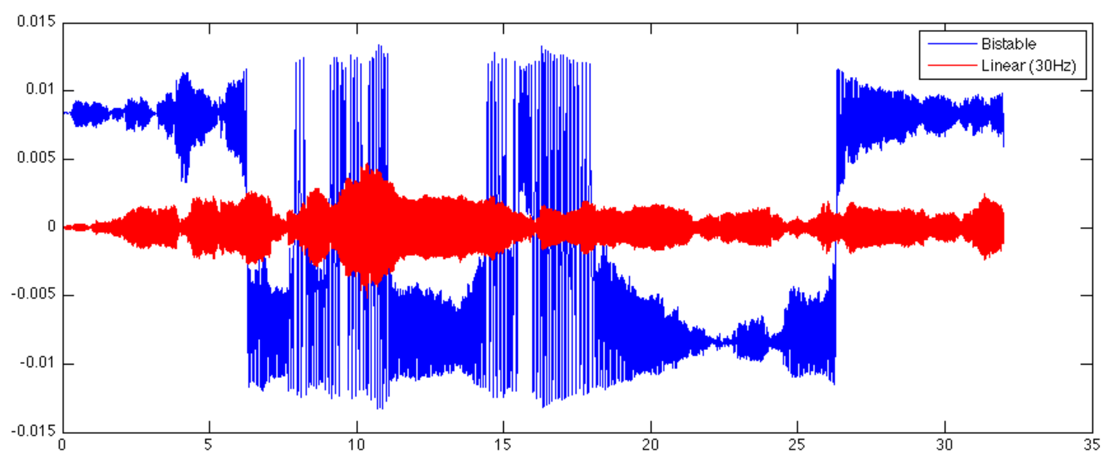
$$b_{MAX} = \frac{a^2}{4D\ \log(\tau_p)}$$

L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

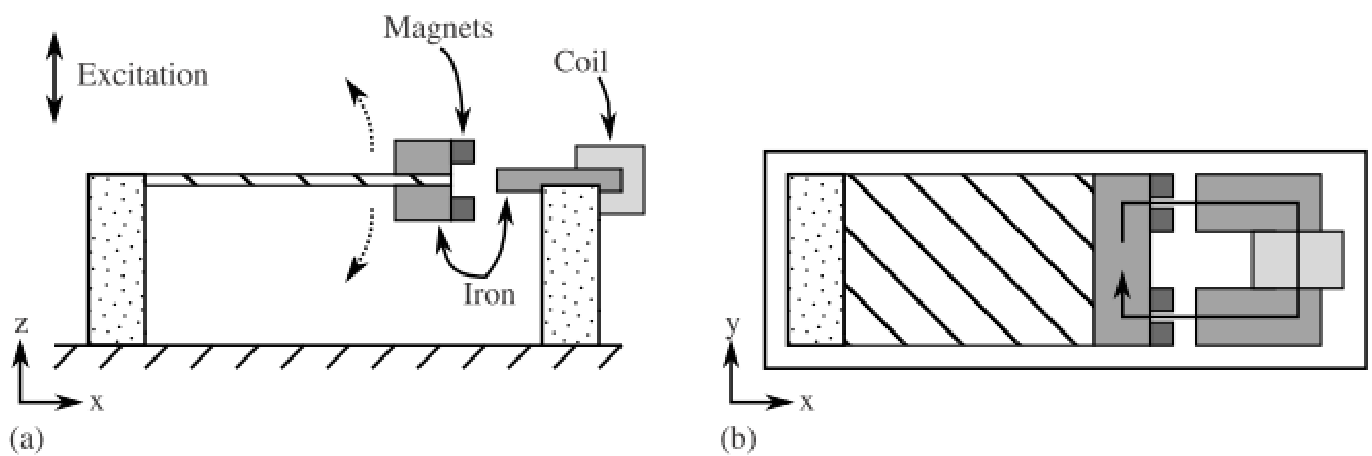
A compared response







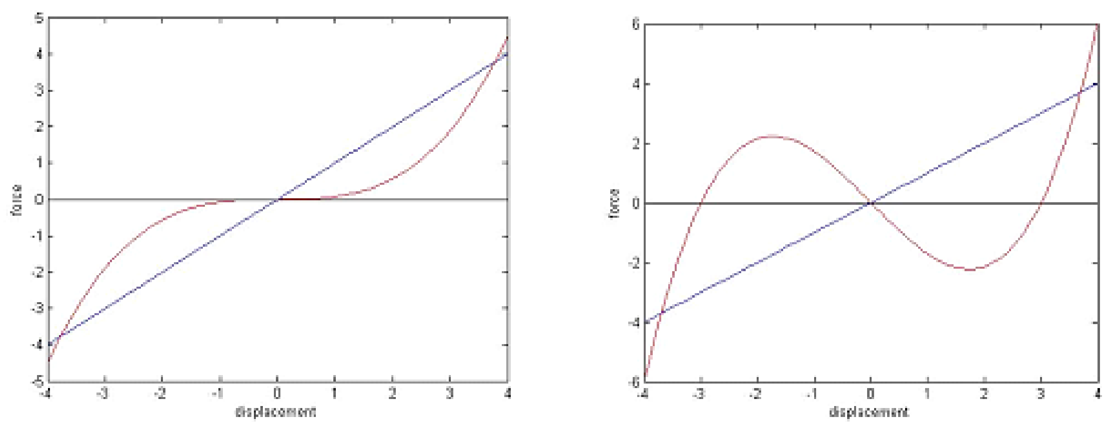
Barton D.A.W., Burrow S.G. and Clare L.R., 2010, “Energy Harvesting from Vibrations with a Nonlinear Oscillator,” *Journal of Vibration and Acoustics*, 132, 021009.



$$m\ddot{x} + \left(c + \frac{\theta^2}{R_C + R_L}\right)\dot{x} + kx + \beta x^3 = F\sin(\omega t)$$

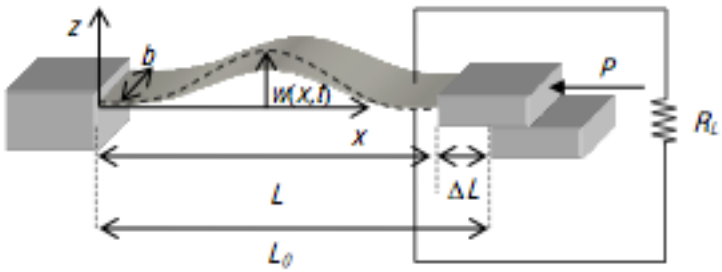
k < *0* → hardening

k > *0* → bistable

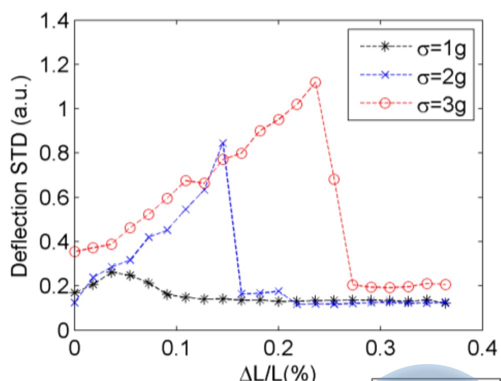


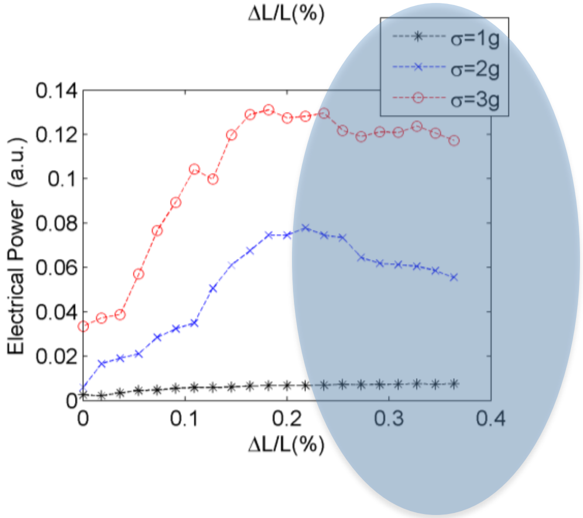
After non-dimensionalization: $\ddot{x} + 2\xi_{eff}\dot{x} + x + \beta x^3 = \Gamma\sin(\omega t)$

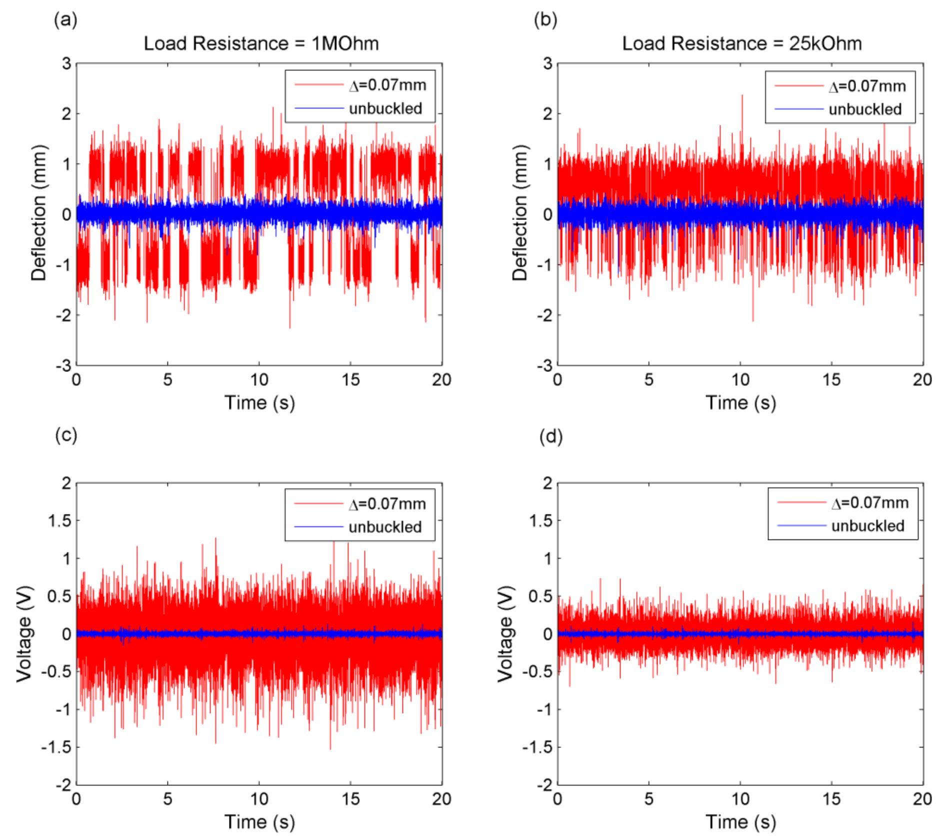
The buckled beam



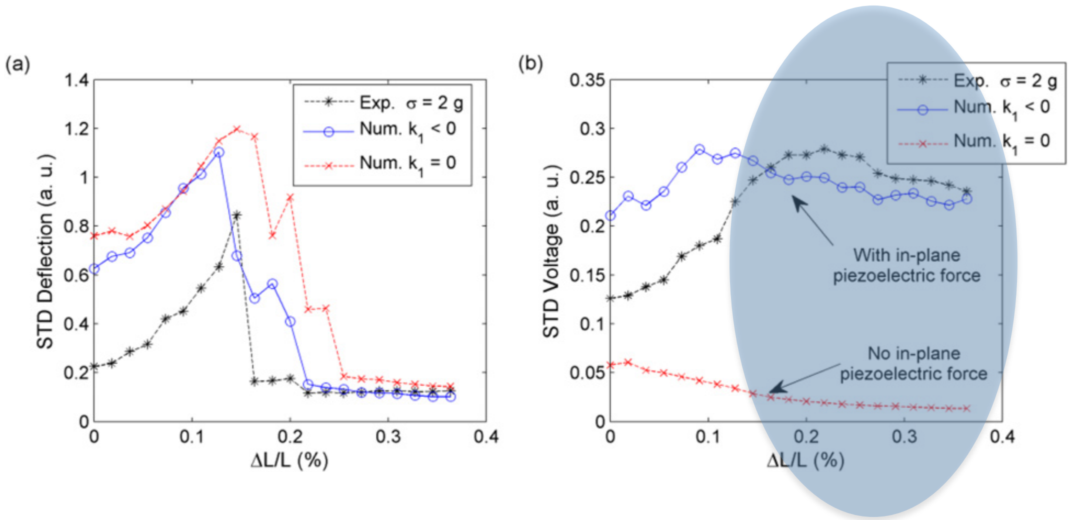
Piezoelectric buckled beams for random vibration energy harvesting
Francesco Cottone, Luca Gammaitoni, Helios Vocca, Marco Ferrari and Vittorio Ferrari
Smart Materials & Structures 21 (2012)







The buckled beam



The governing equations are:

$$m\ddot{x} + \gamma\dot{x} + k_3 x^3 + (k_2 + k_1 V)x - k_0 V = \zeta$$

$$\tfrac{1}{2} C_p \dot{V} + \frac{V}{R_L} = k_1 x\dot{x} - k_0 \dot{x}$$

where k_2 and k_3 are the linear and non-linear stiffness, k_0 is the piezoelectric coupling factor and k_1 is the in-plane piezoelectric force factor.

The conservative force is Duffing-like:

$$U(x) = \tfrac{1}{4} k_3 x^4 + \tfrac{1}{2}(k_2 + k_1 V)x^2$$

where the linear stiffness parameter is a function of the output voltage

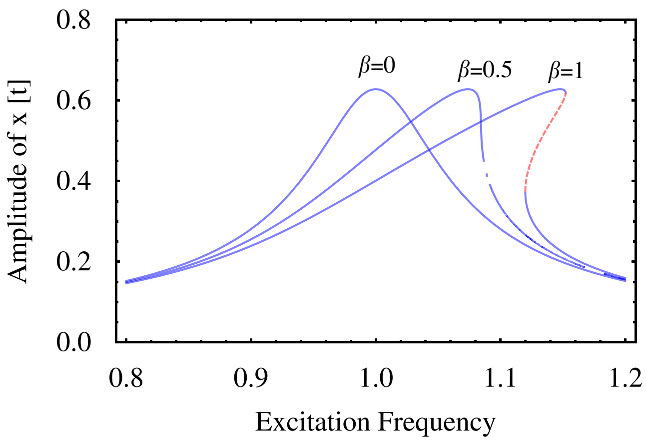
Noise energy harvesting

Only bistability???

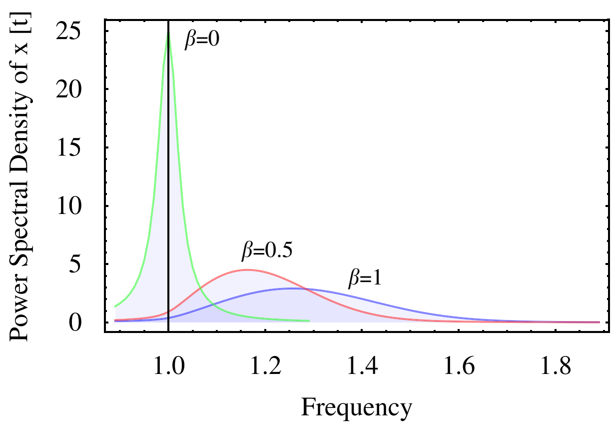
Considering a Duffing oscillator: $\ddot{x} + 2\xi_{eff}\dot{x} + x + \beta x^3 = F(t)$

ξ_{eff} is the effective damping ratio for both electrical and mechanical damping

$\beta > 0$ is a stiffness nonlinearity coefficient



Steady-state frequency response curves under harmonic excitations of a fixed frequency



Power spectral density curves of *x(t)* under White Gaussian excitations of a fixed spectral density.

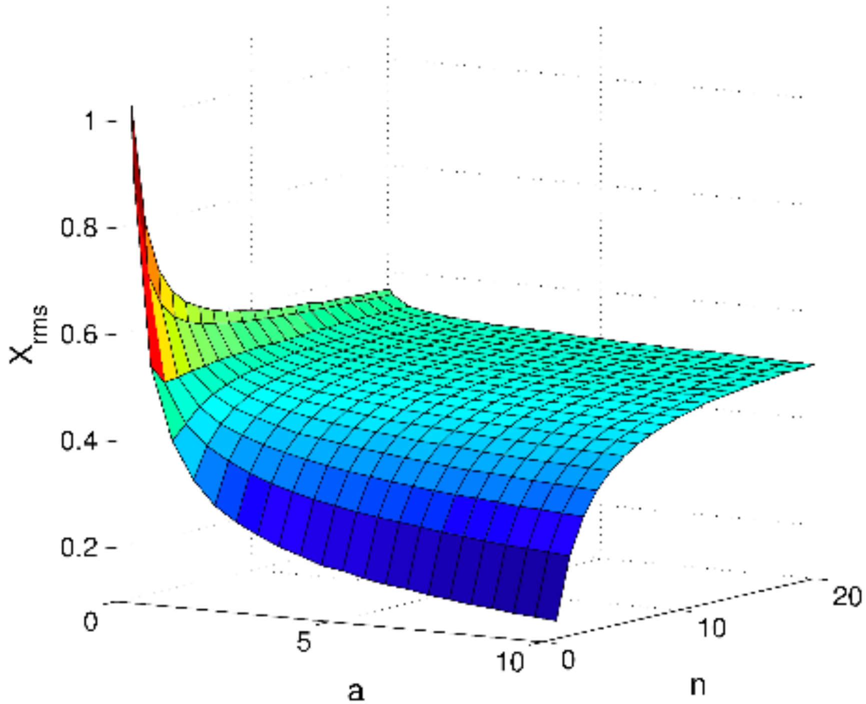
M.F. Daqaq, Journal of Sound and Vibration 329 (2010) 3621–3631

Noise energy harvesting

Only bistability???

A more general monostable potential…

$$U(x) = ax^{2n} \quad \text{with} \quad \begin{array}{l} a > 0 \\ n = 1,2,\ldots \end{array}$$



In an exponentially correlated noise with correlation time τ:

$$\langle \xi(t)\xi(t_1)\rangle = \sigma^2 e^{-\frac{|t-t_1|}{\tau}}$$

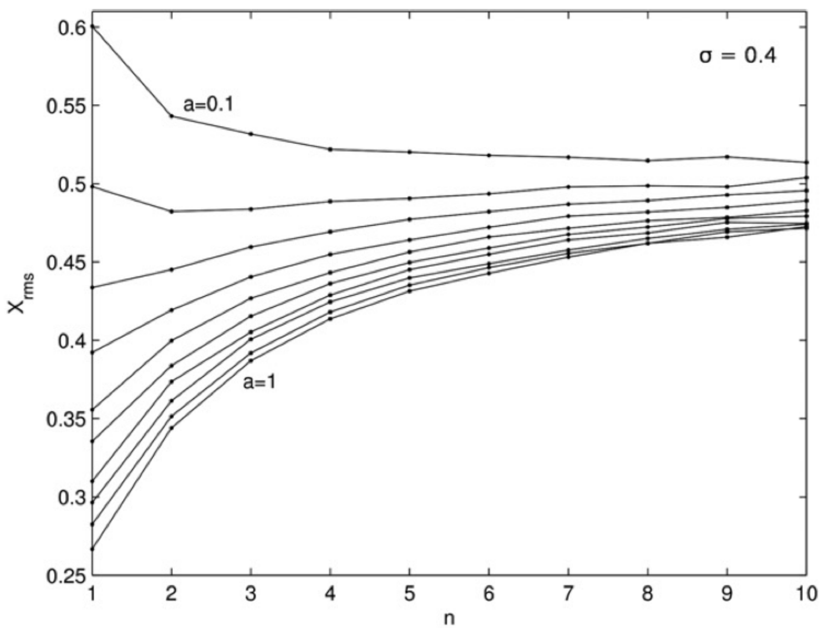
There exists a threshold amplitude a_{th}:

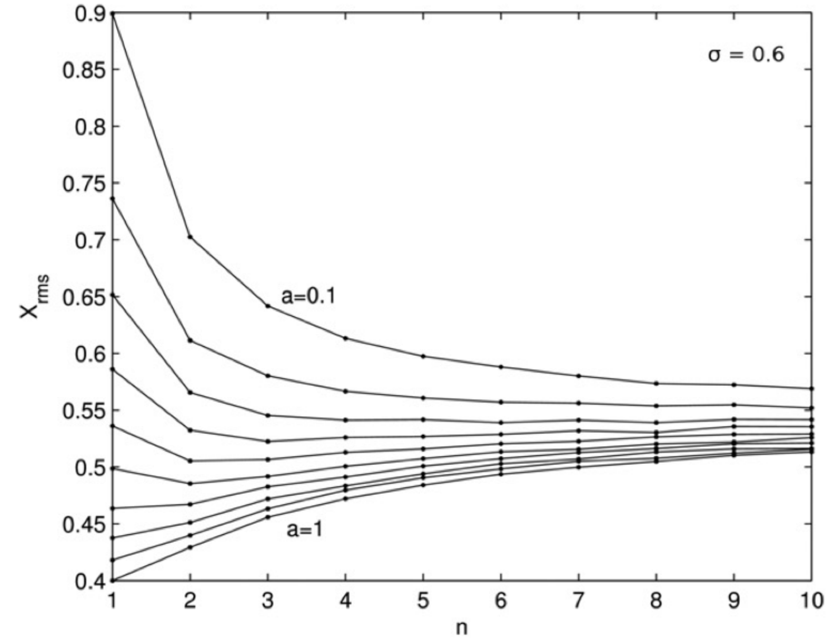
$$a_{th} \approx \frac{D}{4} = \sigma^2\tau$$

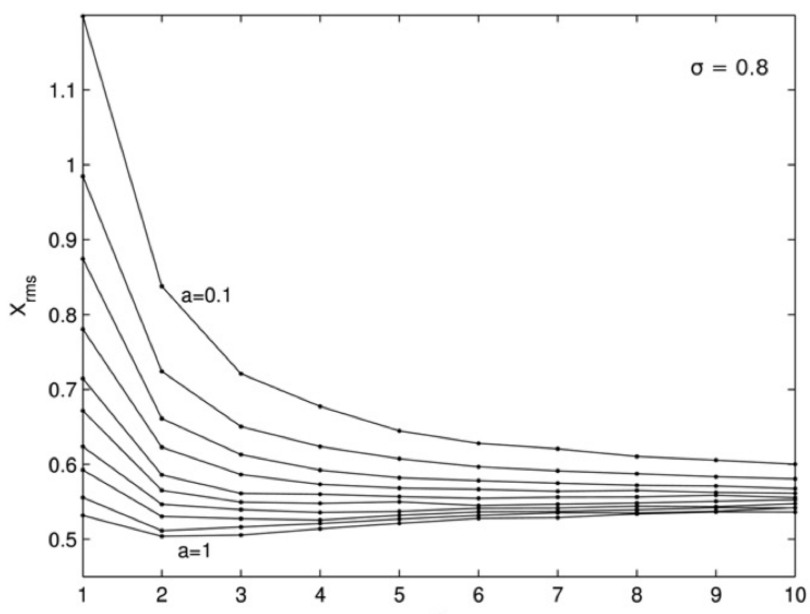
Above which the nonlinear system outperforms the linear one.

L. Gammaitoni, I. Neri, H. Vocca, Appl. Phys. Lett. 94, 164102 (2009)

Varying the noise amplitude

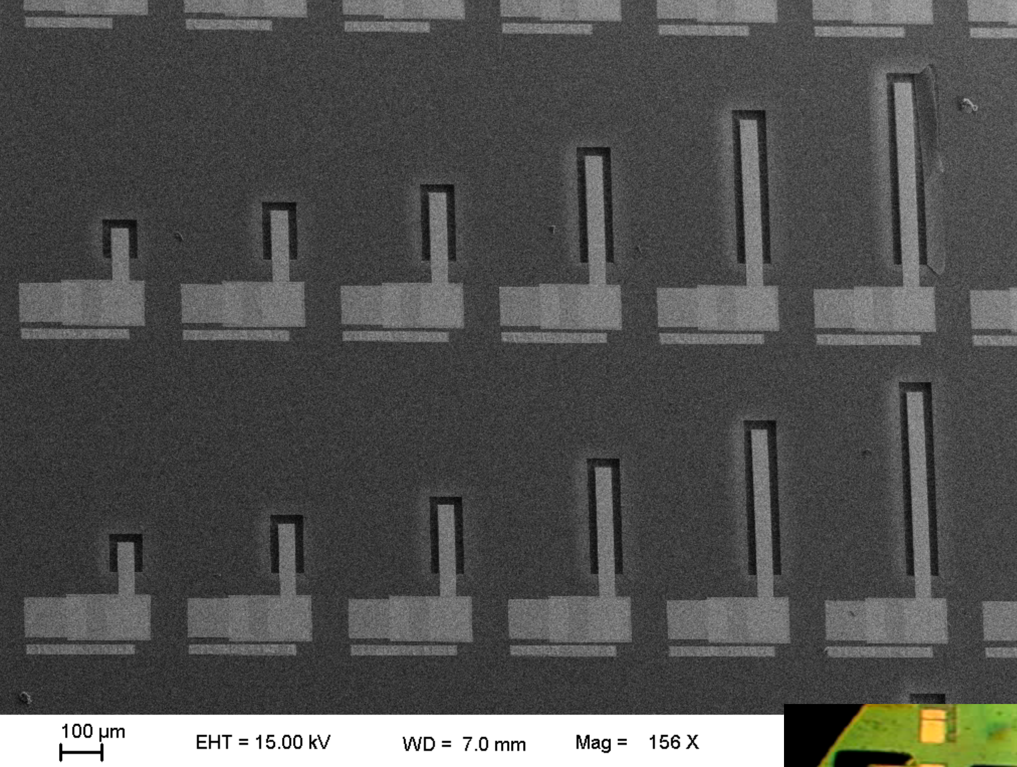




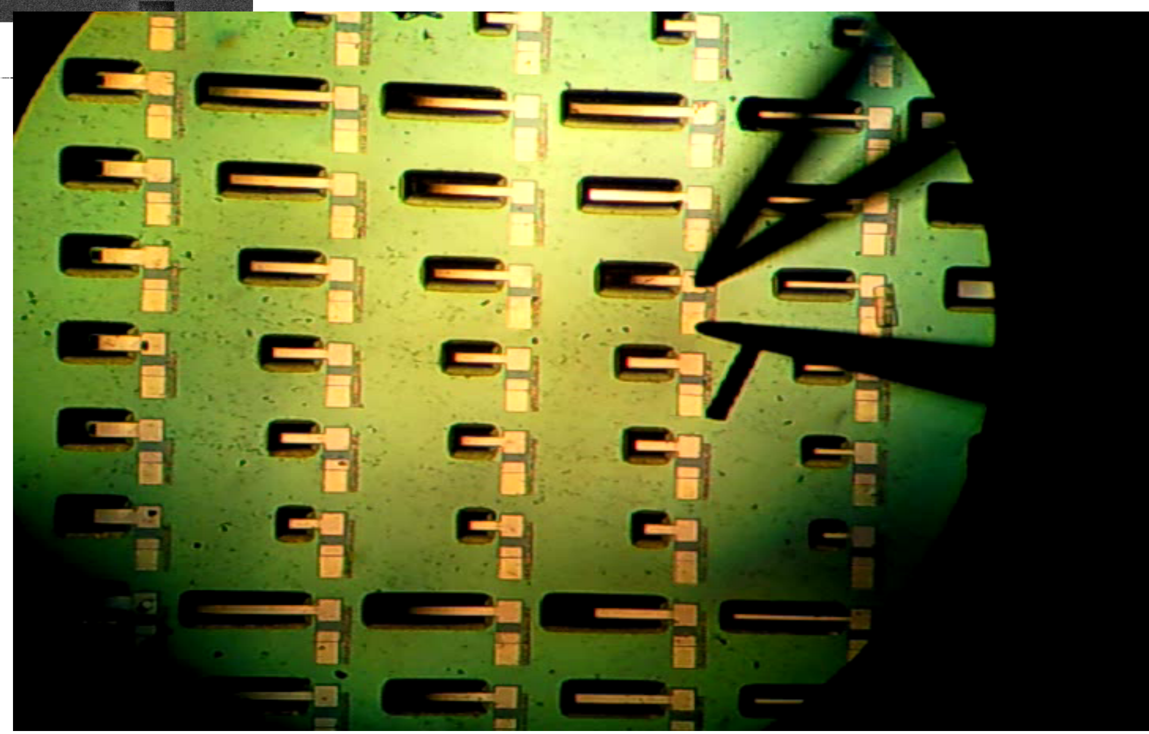


Once **σ** and **a** are fixed the choice of a linear (n = 1) or nonlinear potential (n ≥ 2) can be made in order to maximize $\boldsymbol{x_{rms}}$ and consequently the power obtained at the device output.

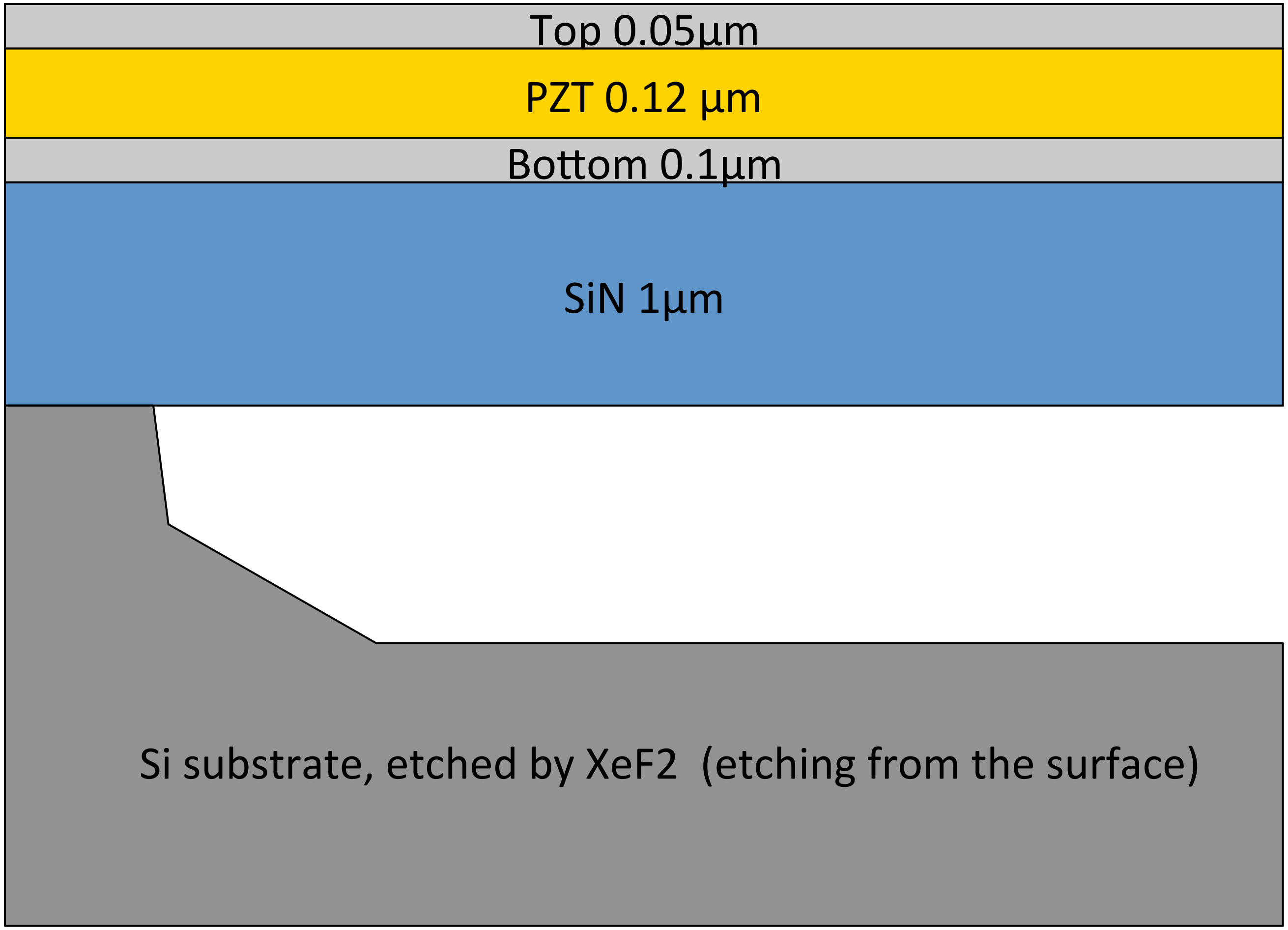
L. Gammaitoni I Neri and H Vocca, Chem. Phys. (2010), doi:10.1016/j.chemphys.2010.08.012



In collaboration with CEA-Leti we are investigating µcantilevers dynamics



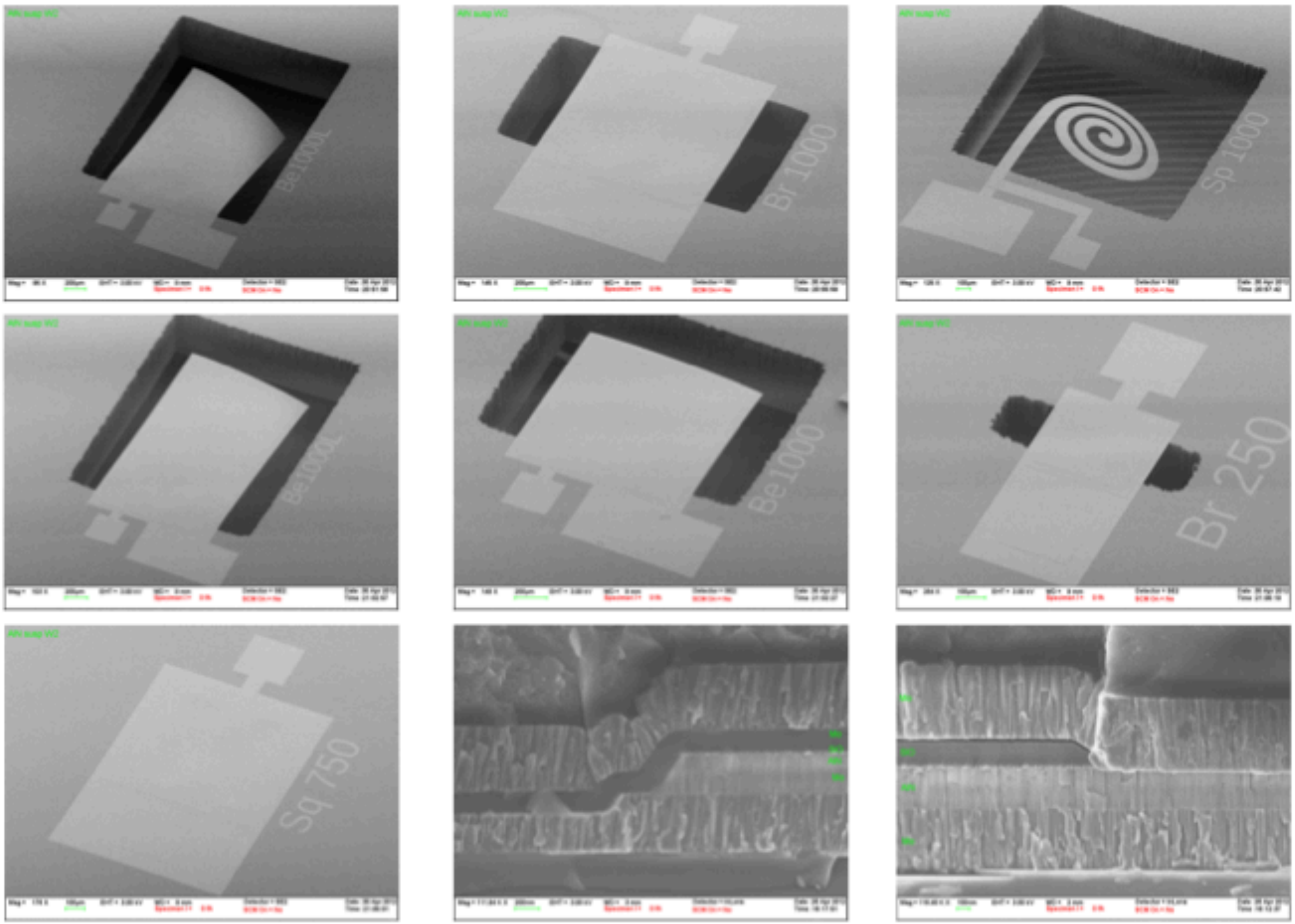
Scheme of the cross section of one cantilever



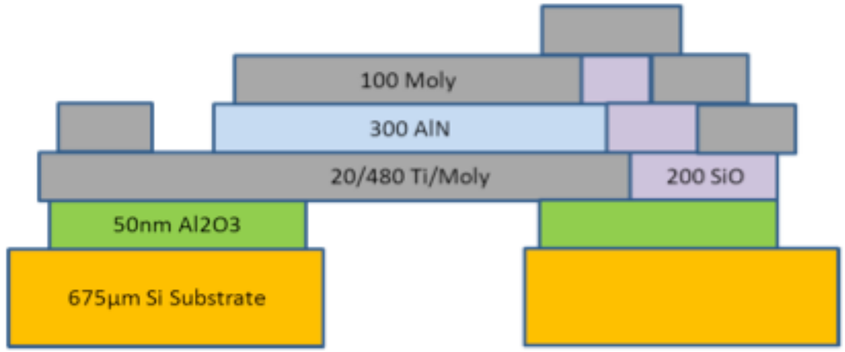
Typical electrical features

- Max voltage sustainable: 5-7V
- e31 PZT = $-5 C/m^2$
- Dielectric constant PZT: 1000

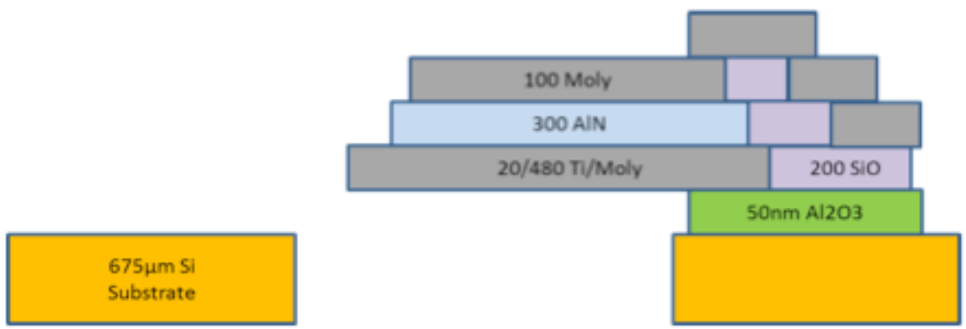
Nonlinear membranes, beams and … from VTT – Helsinki (Fi)



Nonlinear membranes and beams from VTT



Cross section of a membrane harvester shown schematically.



Cross section of a beam device supported from only side.

Statistics for linear systems

- "1D" Statistics: (2nd Order Cumulants, 1st Order Spectra)
  - Correlation: $C_{xy}(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, d\tau \quad \Leftrightarrow \quad X(f)\, Y^*(f) = S_{xy}(f)$
  - Power Spectral Density: $C_{2x}(t) \quad \Leftrightarrow \quad X(f)\, X^*(f) = S_{2x}(f)$
  - Coherence: $\mathrm{C}_{xy}(f) = \dfrac{S_{xy}(f)}{\sqrt{S_{2x}(f)\, S_{2y}(f)}}$
    - Tells us power and phase coherence at a given frequency

Statistics for non-linear systems

- “2D” Statistics: (3rd Order Cumulants, 2nd Order Spectra)

Impossibile visualizzare l'immagine. La memoria del computer potrebbe essere insufficiente per aprire l'immagine oppure l'immagine potrebbe essere danneggiata. Riavviare il computer e aprire di nuovo il file. Se viene visualizzata di nuovo la x rossa, potrebbe essere necessario eliminare l'immagine e inserirla di nuovo.

  – Bicumulant:

$$C_{xyz}(t,t') = \int_{-\infty}^{\infty} x(\tau)\, y(t+\tau)\, z(t'+\tau)\, d\tau \quad \Leftrightarrow \quad X(f_1)\, Y(f_2)\, Z^*(f_1+f_2) = S_{xyz}(f_1,f_2)$$

  – Bispectral Density:

$$C_{3x}(t) \quad \Leftrightarrow \quad X(f_1)\, X(f_2)\, X^*(f_1+f_2) = S_{3x}(f_1,f_2)$$

$$S_{3x}(f_1,f_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} C_{3x}(m,n)\, e^{2\pi i (f_1 m + f_2 n)}\, dm\, dn$$

  – Bicoherence:

$$\mathbf{c}_{xyz}(f) = \frac{S_{xyz}(f_1,f_2)}{\sqrt{S_{xx}(f_1)}\sqrt{S_{yy}(f_2)}\sqrt{S_{zz}(f_1+f_2)}}$$

    - Tells us power and phase coherence at a coupled frequency

Statistics for non-linear systems

The Spectrogram (*STFT square modulus*):

$$S_x(t,\nu) = \left| \int_{-\infty}^{+\infty} x(\tau) h^*(\tau - t) e^{-i2\pi\nu\tau} d\tau \right|^2$$

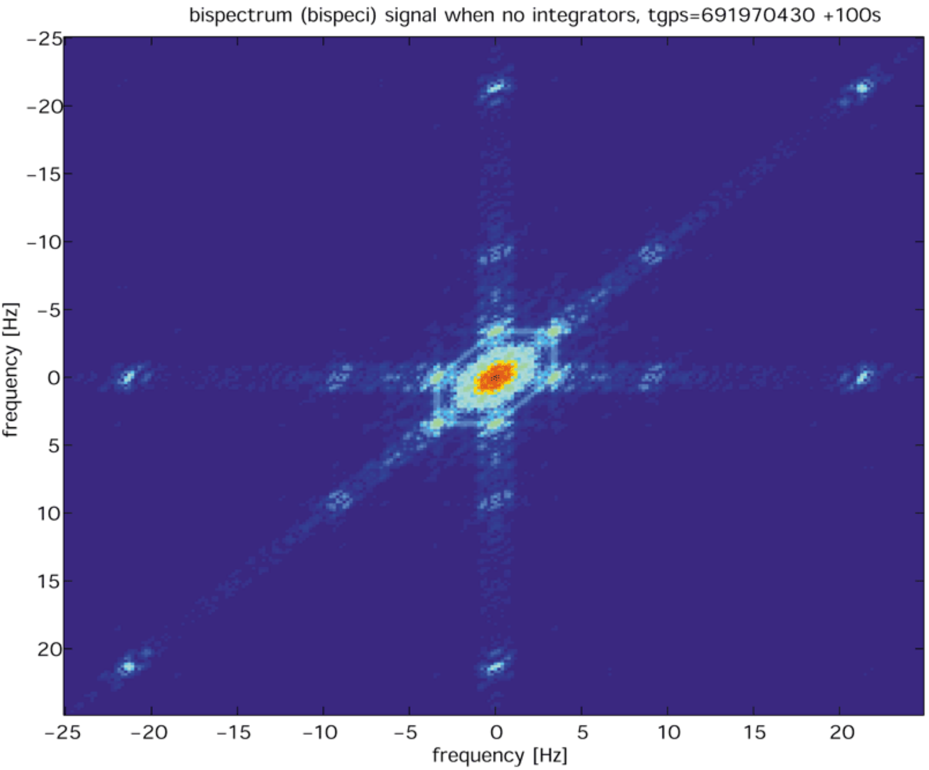
Represents the signal energy in the time-frequency domain centred in (t,ν).

- To analize the system linearity bispectrum and bicoherence need to be taken into account:

- If $S_{3x}=0$ the process is Gaussian and linear

- If $S_{3x} \neq 0$ the process is not Gaussian and

    - if c_{3x} is constant - the process is linear
    - if c_{3x} is not constant - the process is not linear

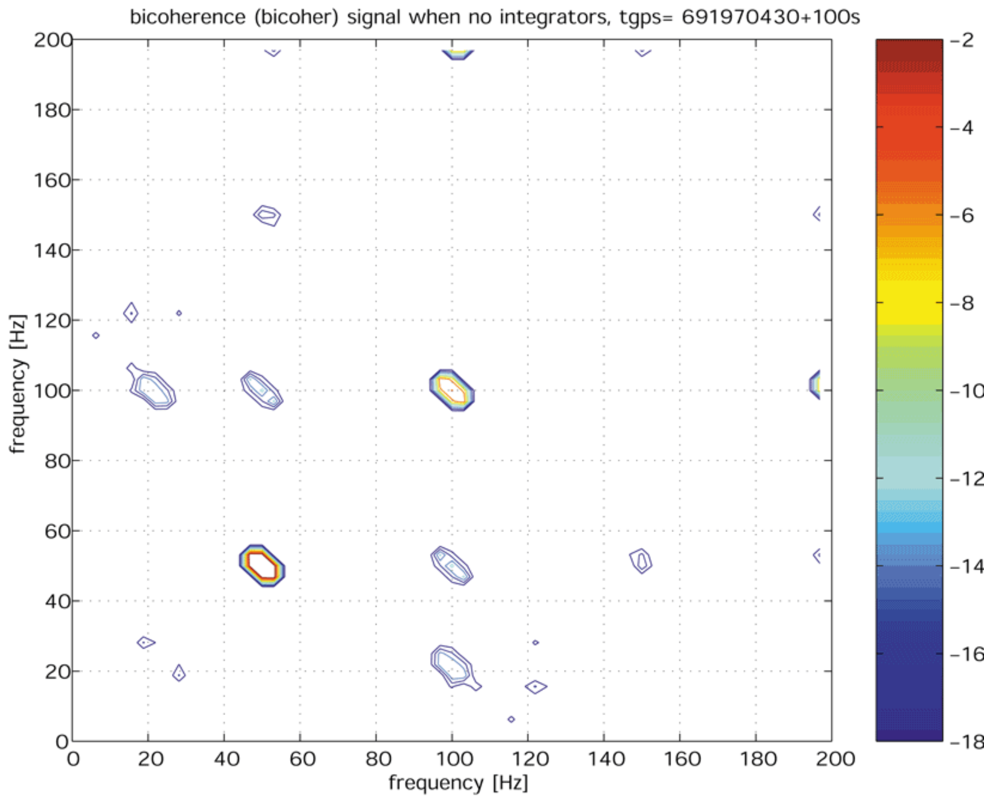


Bispectrum

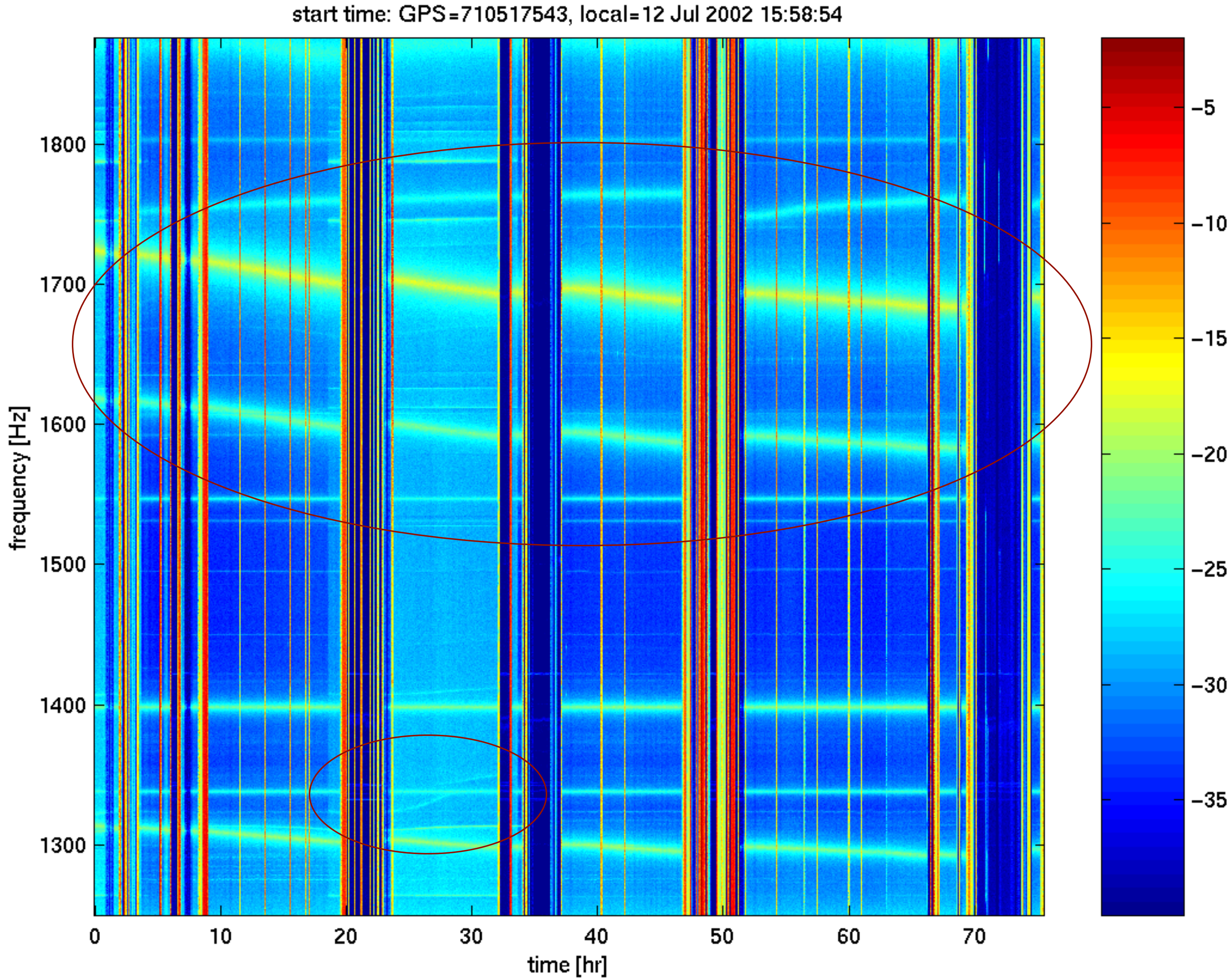
Low frequency noise coupled at higher frequencies

Bicoherence

A nonlinearity of the 50 Hz with its armonics is observed.
There is present a big coupling between the 20 Hz and the 100 Hz and a smaller one between the 20 and 30 Hz.



Spectrogram:



Conclusions

1) Non resonant (i.e. non-linear) mechanical oscillators can outperform resonant (i.e. linear) ones

2) Non-linear systems are more difficult to treat but more interesting…

3) Bistability is not the only nonlinearity available…

4) The same principles are also valid for capacitive and inductive harvesters

5) A great amount of work has still to be done… good for us!!!